Gamma-Ray Imaging

- Introduction to gamma-ray imaging
  - Motivations, goals, applications
- Basic concepts
  - Emission vs. transmission imaging
  - Projective (2D) vs. tomographic (3D) imaging
  - Image quality
    - Angular/spatial resolution, image SNR, contrast
- Modalities
  - Collimator-based
    - Pinhole
    - Regular multi-aperture (parallel-hole, converging/diverging, etc)
    - Coded aperture
  - Collimator-less
    - Kinematic imaging (Compton imaging)
- Image reconstruction

Common Objectives of Imaging

- Recover information about the angular/spatial distribution of object(s) non-invasively
  - **Localization:** locate source or feature in some scene/environment
  - **Characterization:** Spatially & temporally dependent features of object
    - Shape, extent, curvature, intensity differences, etc.
- Improve *detection sensitivity* by limiting space over which background contributes

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- Improve *detection sensitivity* by limiting space over which background contributes
  - Example: Signal-to-background ratio = 1/100
    - Measurement (count or ROI) from non-imaging system:
      - $\mu_{src} = 100$; $\mu_{bgd} = 10000 \rightarrow$ Detection confidence $< 1\sigma$

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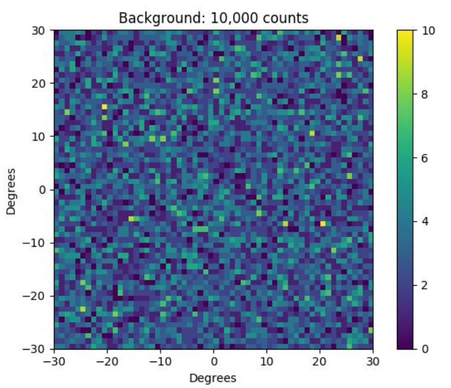
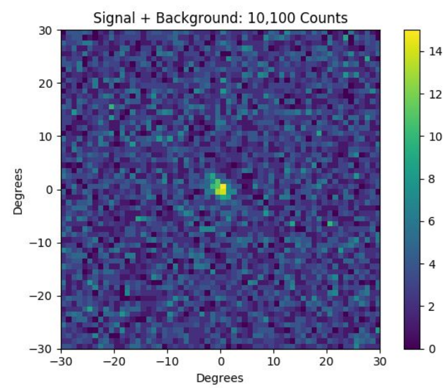
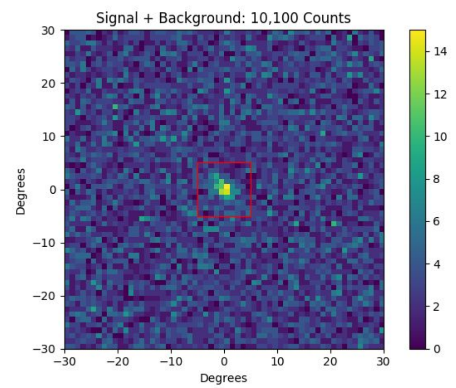
    - Measurement with imaging system





S/B = 100 / 10000 = 0.01



S/B = 100 / 390 = 0.26

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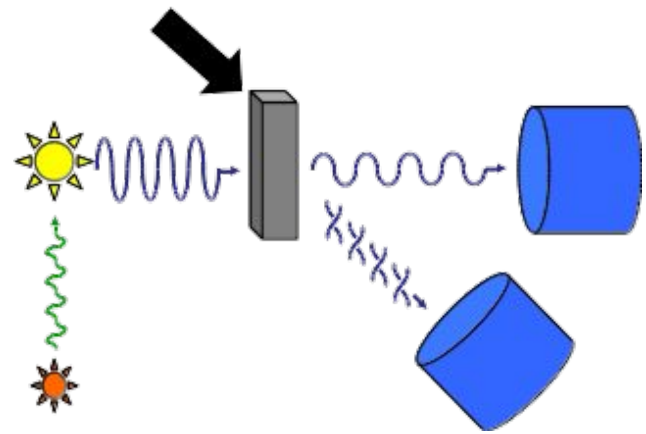
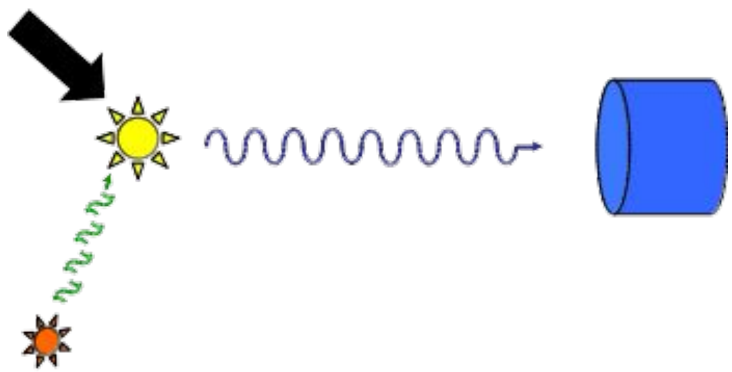
- Improve *detection sensitivity* by limiting space over which background contributes
  - Example: Signal-to-background ratio = 1/100
  - **Caveat!**
    - Imaging capability typically comes with some loss of efficiency!
    - Tradeoff between ***amount*** & ***specificity*** of the information

Transmission Imaging vs. Emission Imaging

- Transmission
  - Gamma-rays to probe object
    - Sensitive to μ, ρ
    - Constrained measurement geometry (emitter & detector)
  - E.g. radiography, CT



- Emission
  - Gamma-rays from decay of radionuclides
  - Relevant for many nuclear security applications
    - Localizing/mapping nuclear material
    - Medical examples: PET, SPECT



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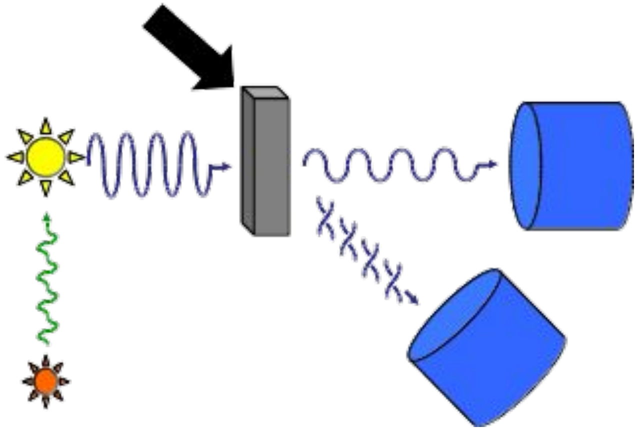

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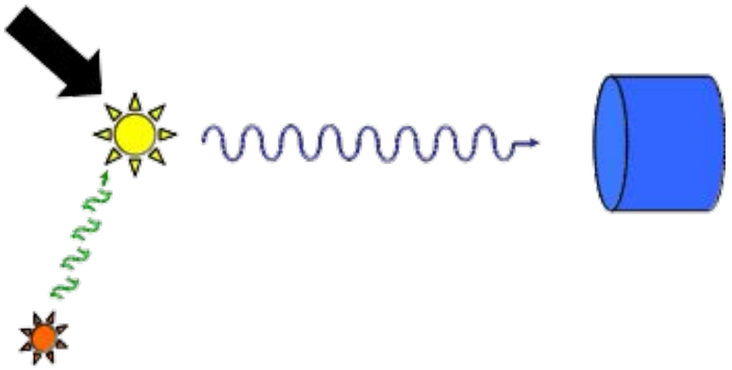
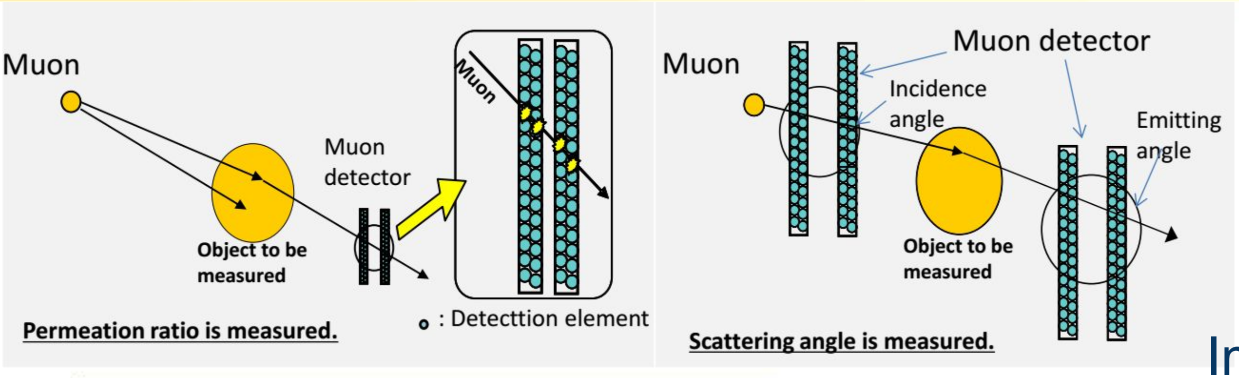
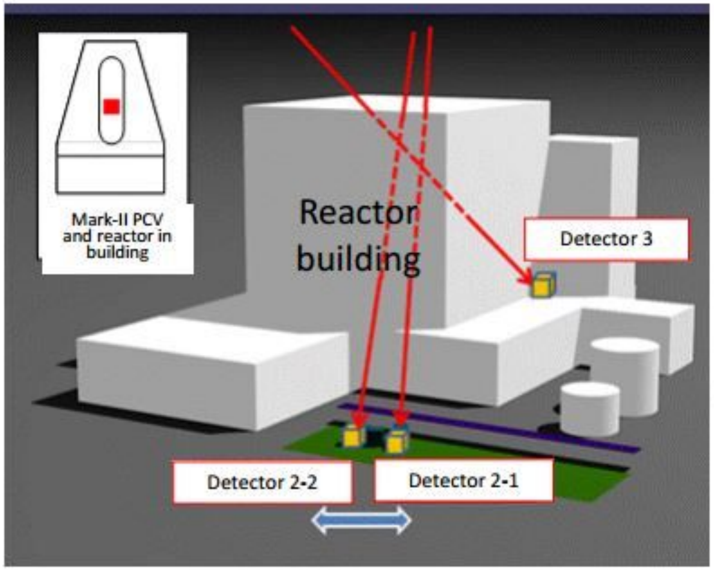


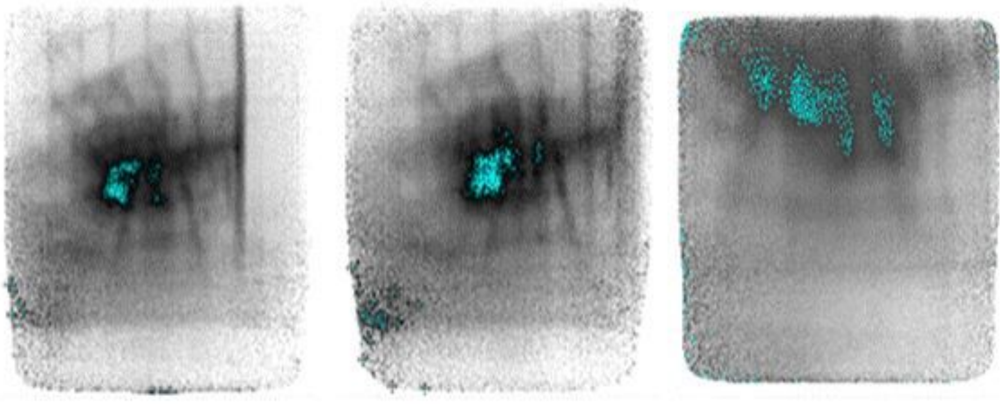
Aside - Transmission Imaging by Muon Tomography



Images from IRID:
http://www.irid.or.jp/_pdf/140914.pdf



Measurement positions at Tokai No. 2 nuclear power plant: (3 points)



Detector 2-1 Detector 2-2 Detector 3

Measurement result at each point
High-density substance existed outside frustum-shaped PCV

KEK(High Energy Acceleration Research Organization)data

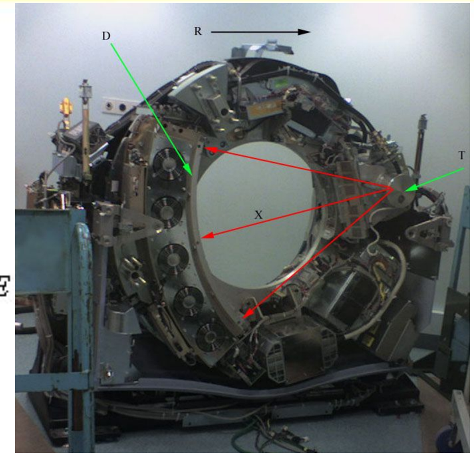
Applications of Gamma-Ray Imaging

- Biomedical imaging
  - Diagnostic imaging, theranostics, beam verification
  - Pharmaceutical development/evaluation (small-animal models)
- Physics, Biology, Chemistry
  - Metabolism and biological/ecological transport
  - Beam diagnostics
- Astrophysics
  - Satellite & balloon-born gamma-ray imagers
  - Stellar/galactic structure, GRB's
- Nuclear security & safeguards
  - Source search
  - Treaty verification
  - Portal & nuclear facility monitoring, dose planning
- Environmental monitoring
  - Consequence management
  - Nuclear contamination remediation

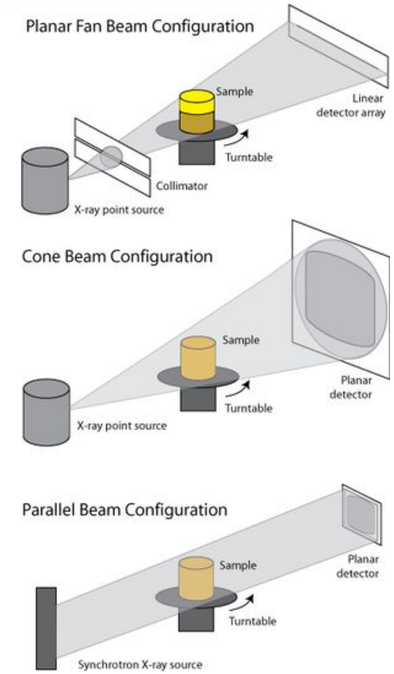
N.B. A very incomplete listing!

Medical Applications: Diagnostic Imaging

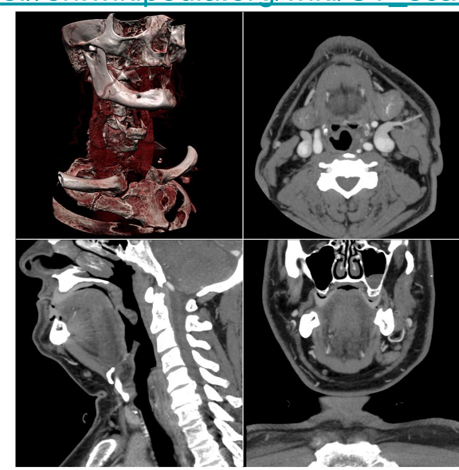
- Anatomical/structural imaging
  - Transmission-based
  - X-ray CT
    - Signal basis: $I = \int I_0(E) \exp\left[\sum_i (-\mu_i(E) x_i)\right] dE$
    - Often combined with PET/SPECT
      - Anatomical reference
      - Attenuation correction
  - Instrumentation
    - CsI(Tl) w/ Si photodiode
      - Recall: SiPD - no internal gain
    - Current-mode operation
- Other X-ray imaging techniques
  - Radiography, X-ray fluoroscopy, mammography, x-ray angiography



https://en.wikipedia.org/wiki/CT_scan



https://serc.carleton.edu/msu_nanotech/methods/CT.html



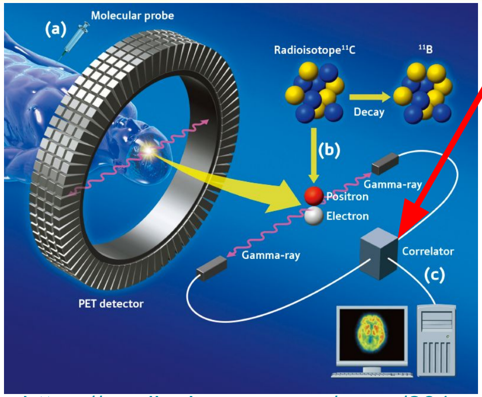
Multi-planar view of CT reconstruction w/ volume rendering Wikipedia: attr ChumpusRex

$$\lambda_\phi(x') = -\ln\left[\frac{I_\phi(x')}{I_\phi^0(x')}\right] = \iint_{-\infty}^{\infty} \mu[x,y]\,\delta(x\cos\phi + y\sin\phi - x')\,dx\,dy$$

B. Camanzi Lec. 3: X-ray Imaging

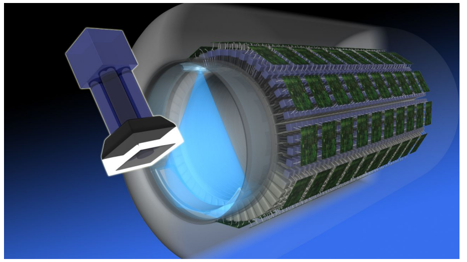
Medical Applications: Diagnostic Imaging

- Diagnostic nuclear medicine
  - Emission-based modalities
  - PET
    - ^{18}FDG very common: proxy for metabolic uptake
    - Instrumentation: Fast scint. (e.g. LSO), long, thin crystals, APD readout
  - SPECT
    - Tomographic collimated imaging
    - ^{99m}Tc (141 keV gamma)-based radiotracer most common
    - Bone scans, perfusion studies (brain, heart), etc.

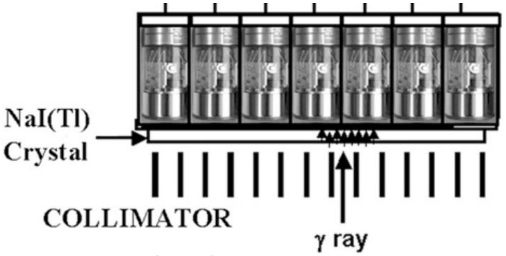


https://medicalxpress.com/news/2010-09-molecular-imaging-vast-world-neuroscience.html

HW very important, see: openPET



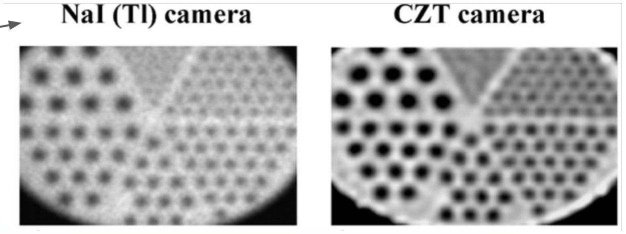
An engineer's dream: UC Davis Explorer - Whole-body PET



Traditional SPECT camera: Gamma-camera based on NaI(Tl) w/ application-specific collimator; multi-PMT readout for pos. sens.

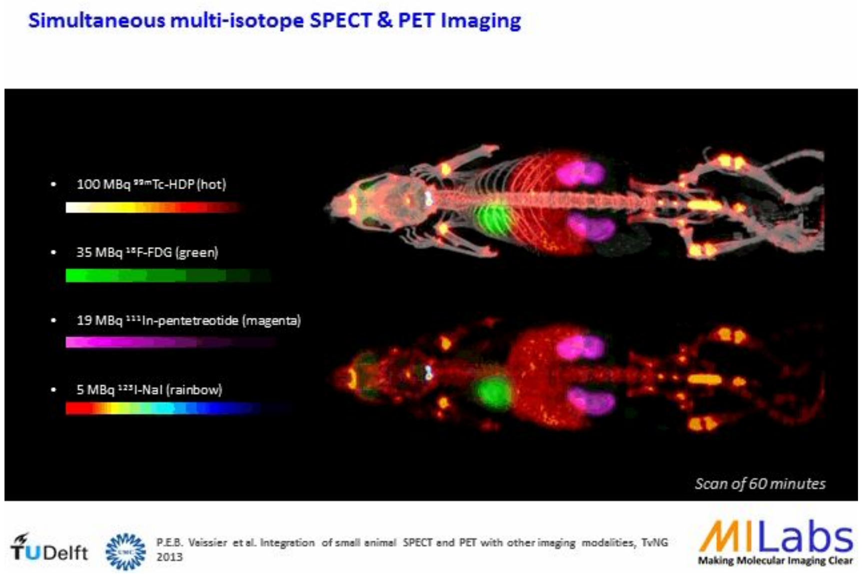
CZT-based SPECT camera - segmented electrodes for pos. Sens. more sensitive, better En. Res. Image from Kromek

SPECT image comparison - comparable resolution, better **contrast** from CZT (better energy resolution → scatter rejection). From Verger et al 2004



Medical Applications: Small-Animal Models

- Pre-clinical imaging applications
    - Pharmaceutical development & evaluation
        - Nice overview here
    - Evaluation of imaging modalities
    - PET & SPECT (often with μ or "micro" prefix)
        - PET/CT, SPECT/CT
- Instrumentation same as clinical case, but smaller scale

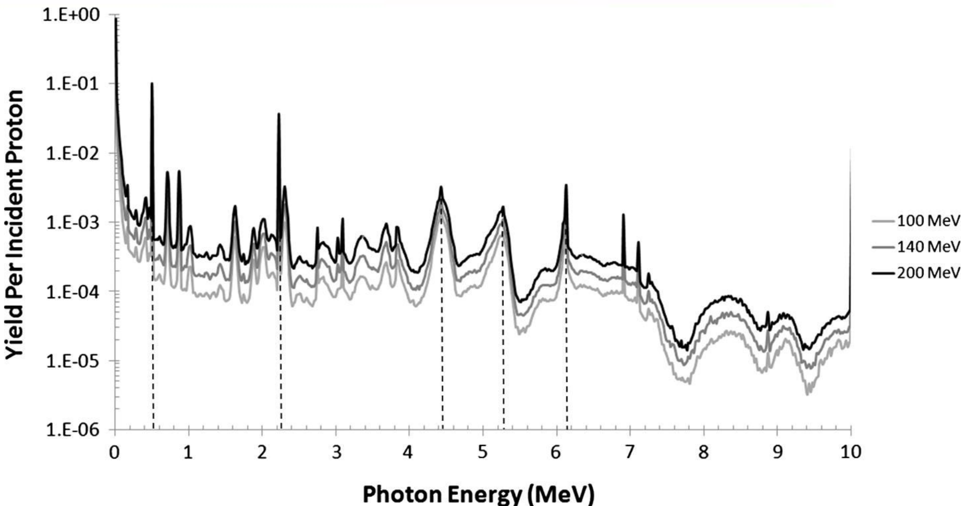


https://www.milabs.com/image-gallery/#PreclinicalGallery

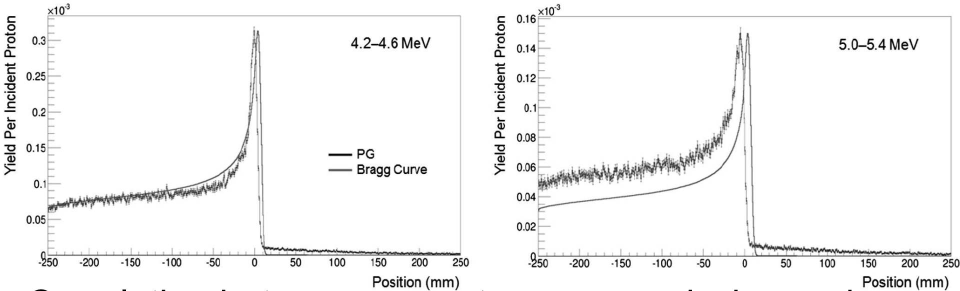
E.g. Nanoscan commercial small-animal PET/CT. Image from JHU

Medical Applications: Beam Verification

- E.g. Proton/Hadron beam cancer therapy
  - Verification of longitudinal dose delivery
  - Gamma-ray imaging based on prompt-gammas from nuclear de-excitations resulting from (p,Nucleus) reactions

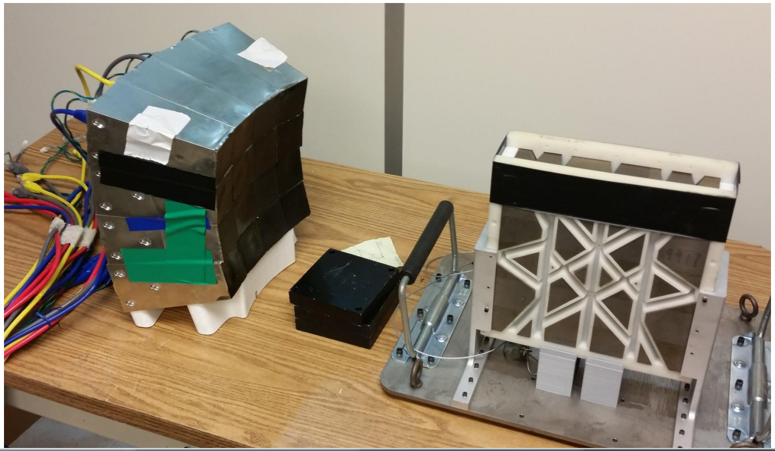


Simulated gamma-ray spectrum from proton pencil beam in H_2O



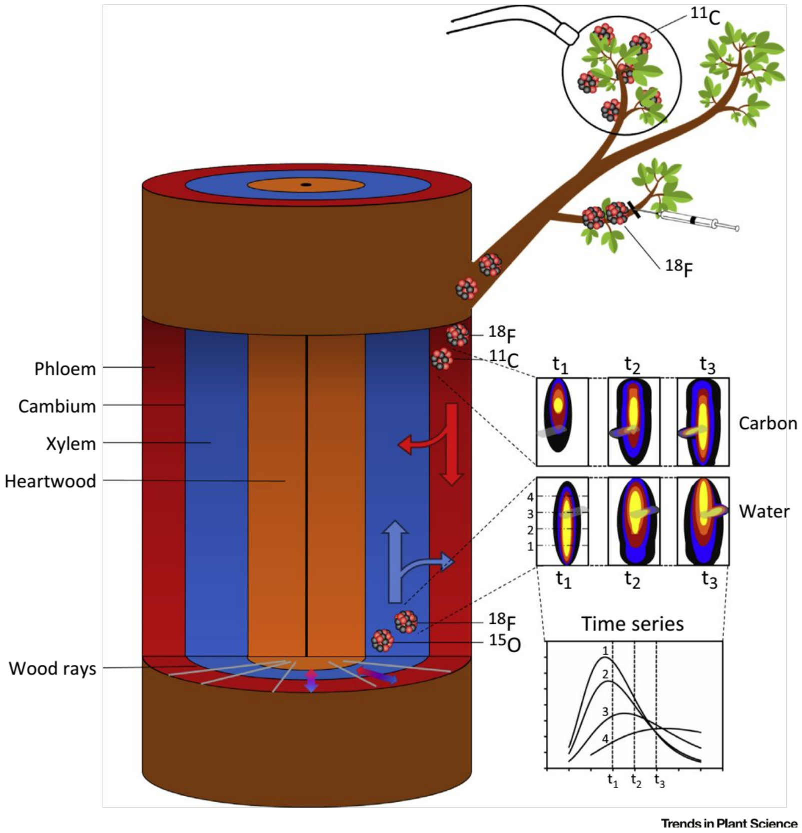
Correlation between prompt-gamma emission and Bragg peak for 200 MeV proton pencil beam (sim.)

Above images from [Zarafiri et. al. 2017]

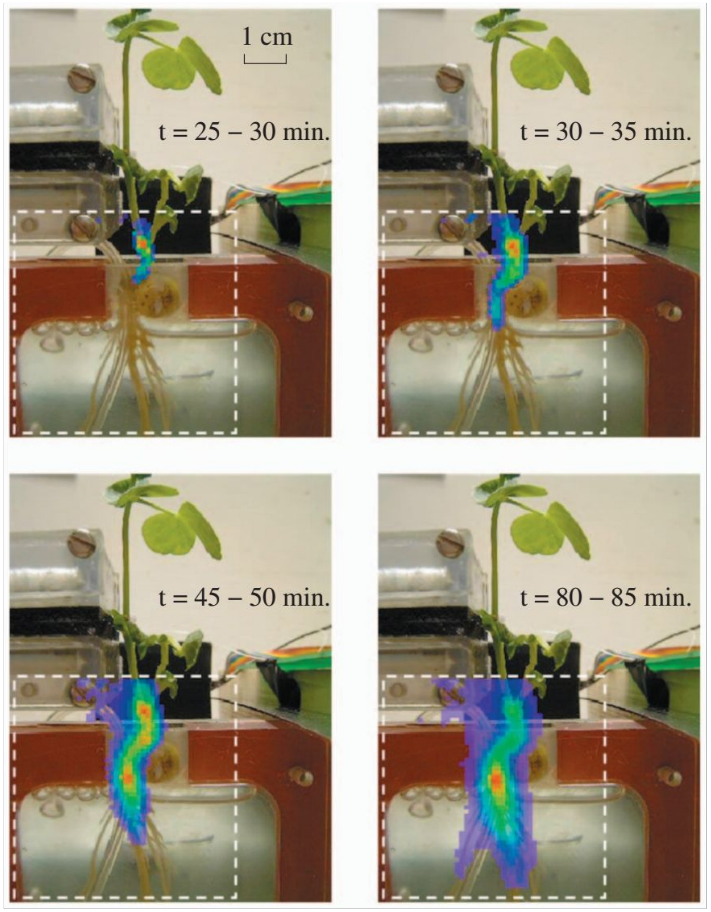


Applications in Biology and Chemistry

- Emission tomography not only for medicine!
  - PET, SPECT, other emission modalities for metabolite uptake & transport



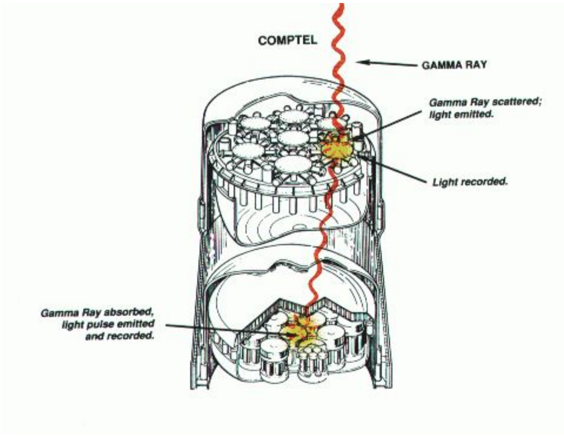
[Hubeau & Steppe, 2015](#)



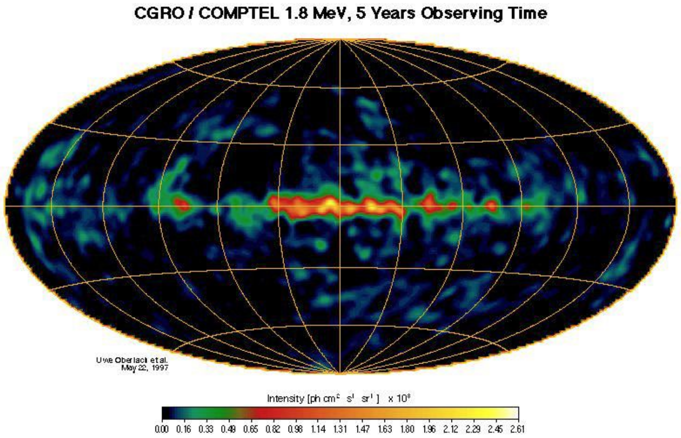
[Kiser et. al. 2018](#)

Space Applications: Astrophysics

Satellite-born gamma-ray imagers



http://cosi.ssl.berkeley.edu/instrument/design/

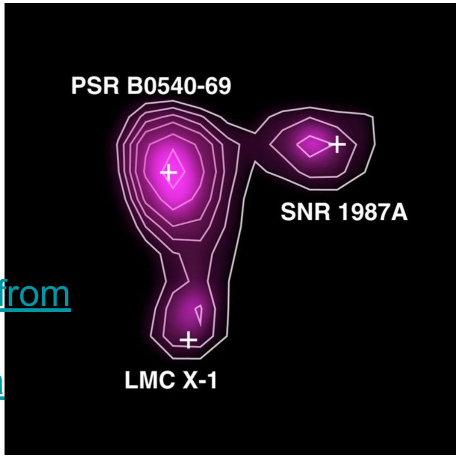


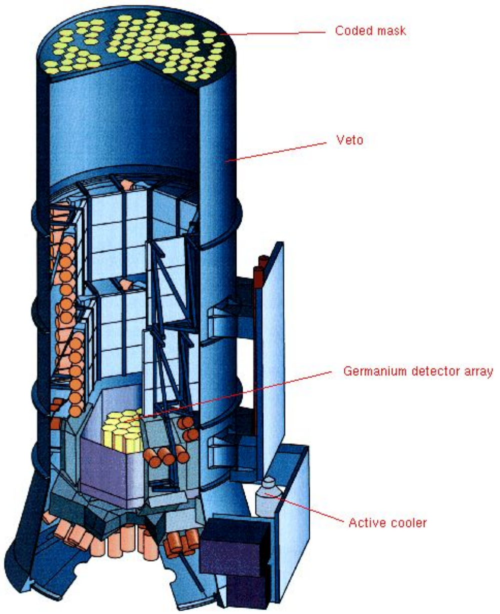
https://heasarc.gsfc.nasa.gov/docs/cgro/comptel/

COMPTEL - Compton imager on CRGO

SPI coded mask

^{44}Ti in remnants from 1987A supernova



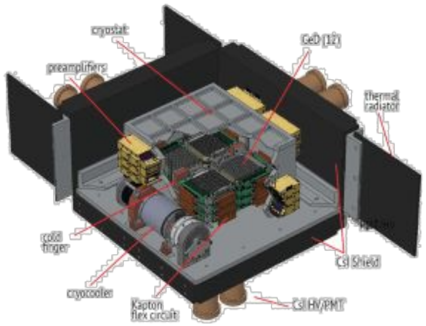


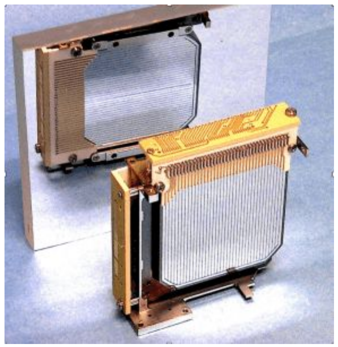
Integral: SPI - HPGe-based Coded-aperture imager

Space Applications: Astrophysics

Balloon-born Gamma-ray imagers

- COSI (formerly NCT) - 12x DSSD HPGe (same as CCI-2)





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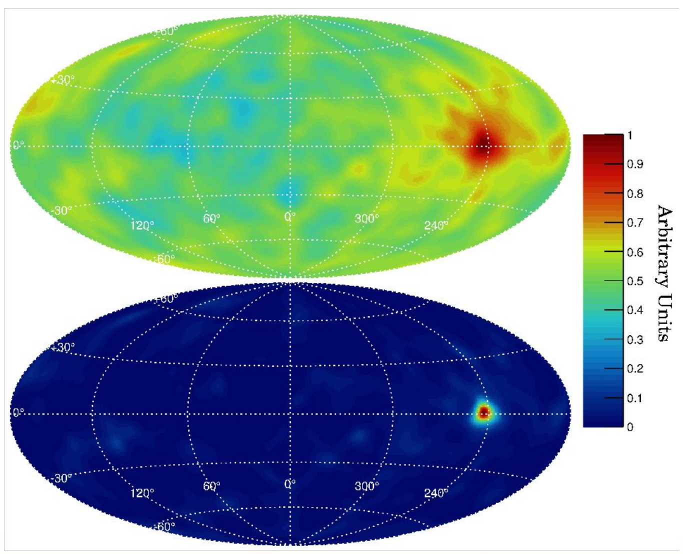
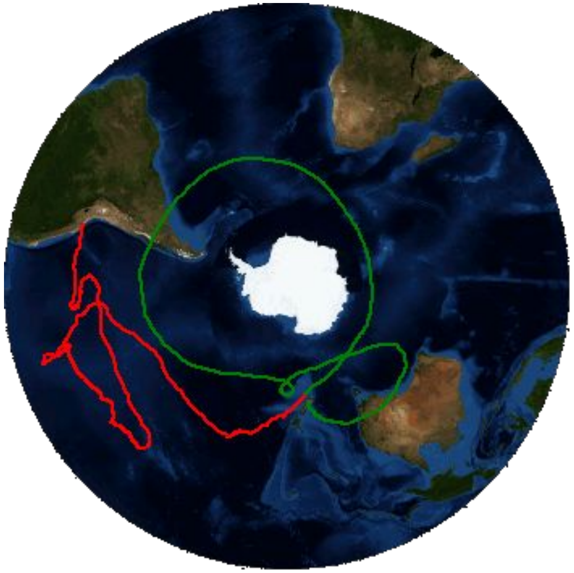


Compton Image localizing gamma-ray burst detected during 2016 flight. Image from Lowell et al, 2017



2016 measurement campaign - balloon liftoff from NZ, landing in Atacama desert

Space Applications: Gamma-ray lenses!

- Laue lens - narrow FOV, focused gamma-ray imaging

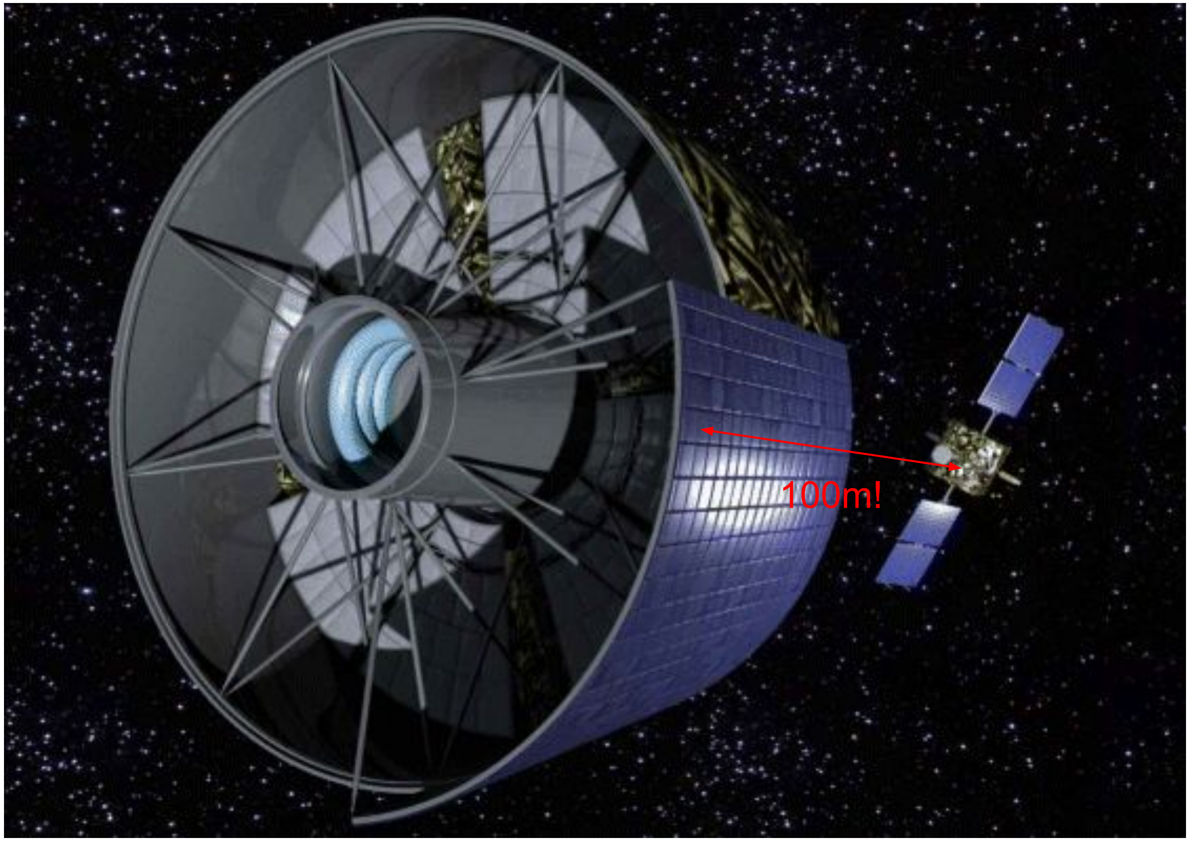
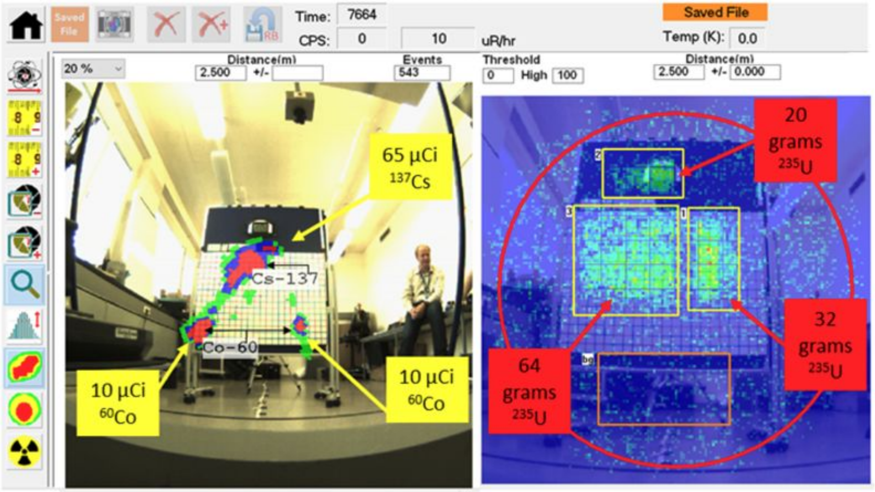


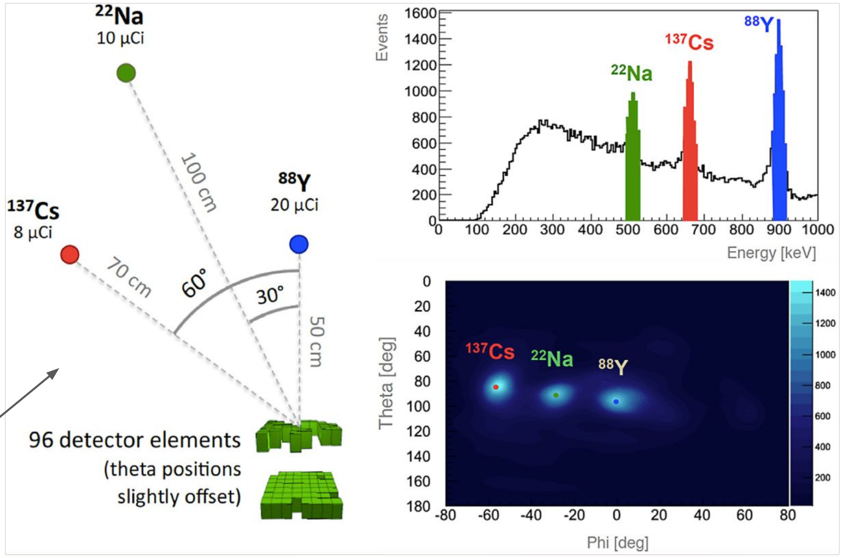
Image from Knodlseder et al, 2009
Other examples of Laue-Lens Telescopes: CLAIRE, DUAL

Nuclear Security and Safeguards

- Detecting, localizing, characterizing nuclear sources
  - Emission imaging
  - Often spectroscopic imaging
- Application drives imaging considerations
  - Source search: wide FOV
  - Characterization
    - Smaller FOV
    - Improved image resolution



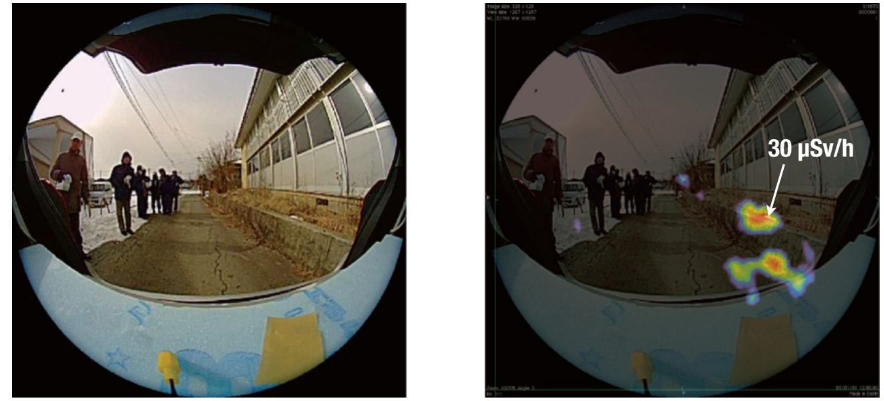
http://www.phdsco.com/



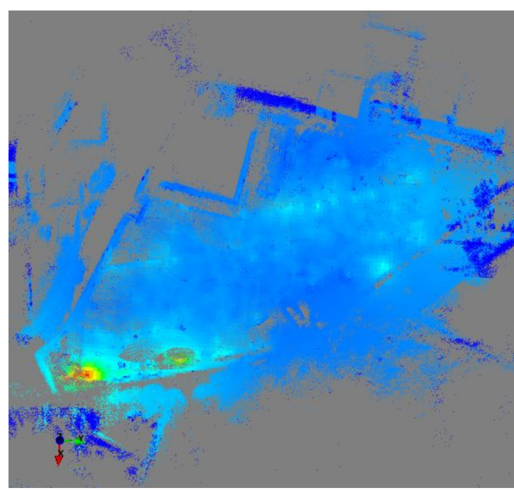
2D Spectroscopic imaging with HEMI

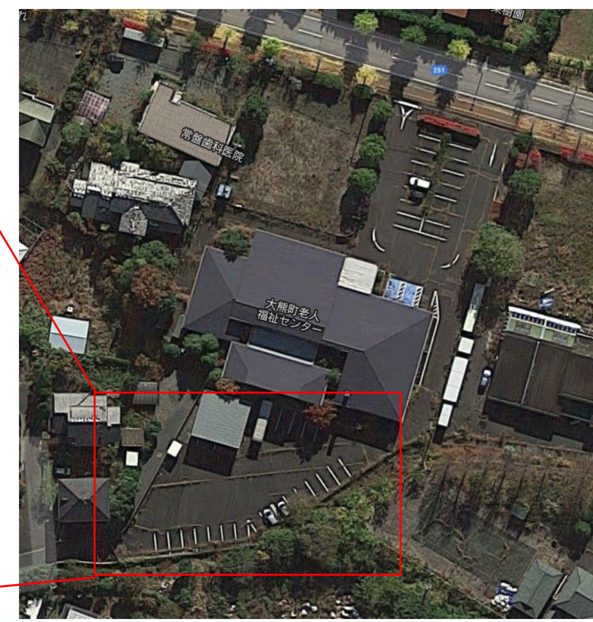
Nuclear Contamination Remediation

- Accurately and efficiently image radionuclide distributions in environment
  - Complex (i.e. non-point) source distributions
  - Large-scale & varied measurement environments
    - Indoor & outdoor
    - 100x100x10 m^3



2D Compton image of radiocesium made with Si/CdTe instrument near Fukushima. From T. Takahashi et. al. 2012





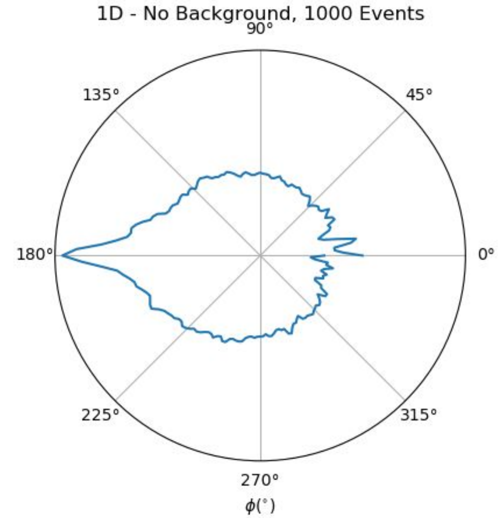
3D Compton image of ^{137}Cs distribution: 15 min measurement with HEMI, March 2017

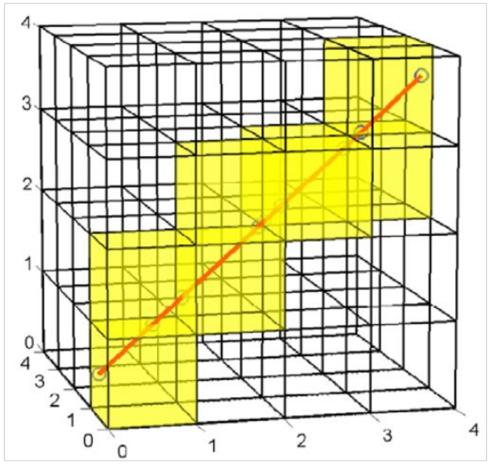
Important concepts for gamma-ray imaging

- Image space
  - Dimensionality
  - Discrete or continuous
- Imaging field
  - Near field vs. far field imaging
  - Point-detector approximation
- Field-of-view
  - Relationship to magnification in nearfield
- Imaging efficiency
  - Signal loss inherent in attenuation-based (collimated) systems
  - Losses due to increased processing requirements
    - Position-sensitivity; gamma-ray event reconstruction
    - Important consideration for Compton imaging

Image Space

- Coordinatization of measurement environment
  - Often discretized
    - Bins (1D), pixels (2D), voxels (3D)
- Image reconstruction involves relating data acquired in ***measurement space*** back to the ***image space***





Line-of-response intersect a **voxellized** image space. Image from [P.C. Hanaen: Regularization in Tomography]

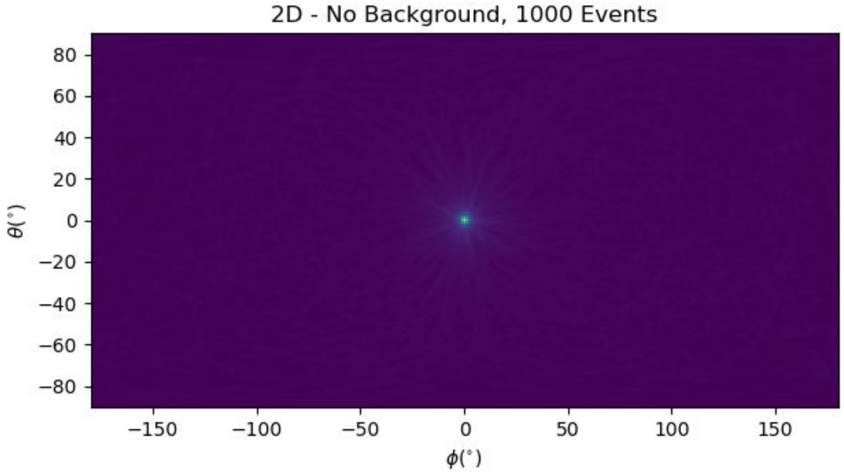
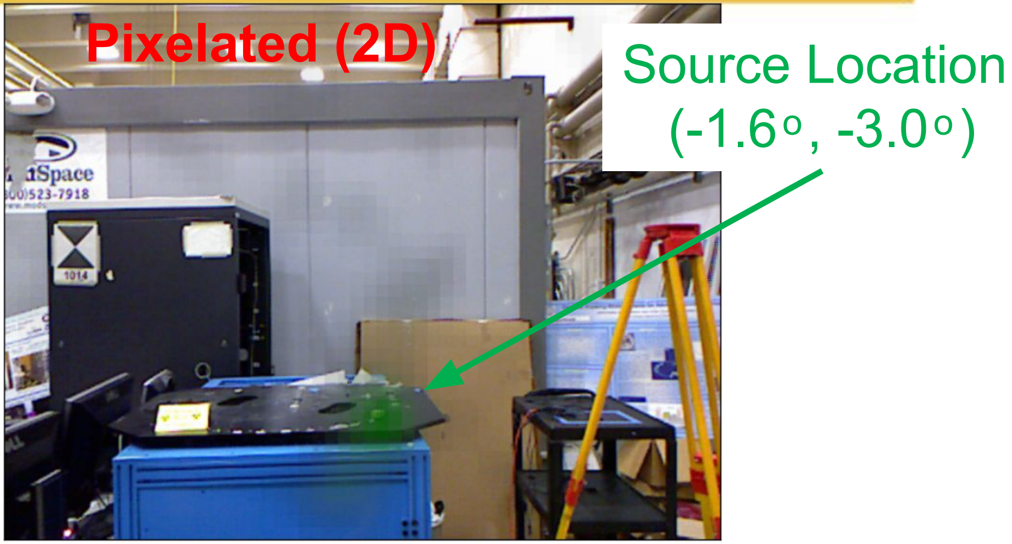
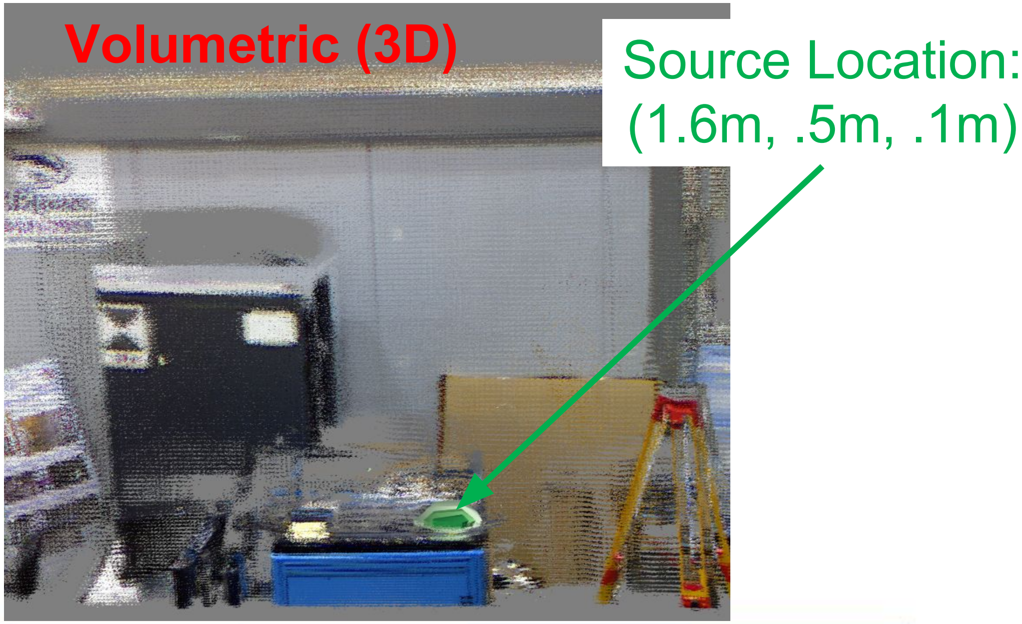


Image Space & Overlays

- In many applications, it is common to fuse the radiation image with another type of image
  - **E.g. conventional image or 3D model**
  - In medicine: molecular & anatomic images (e.g. SPECT/CT, PET/CT)





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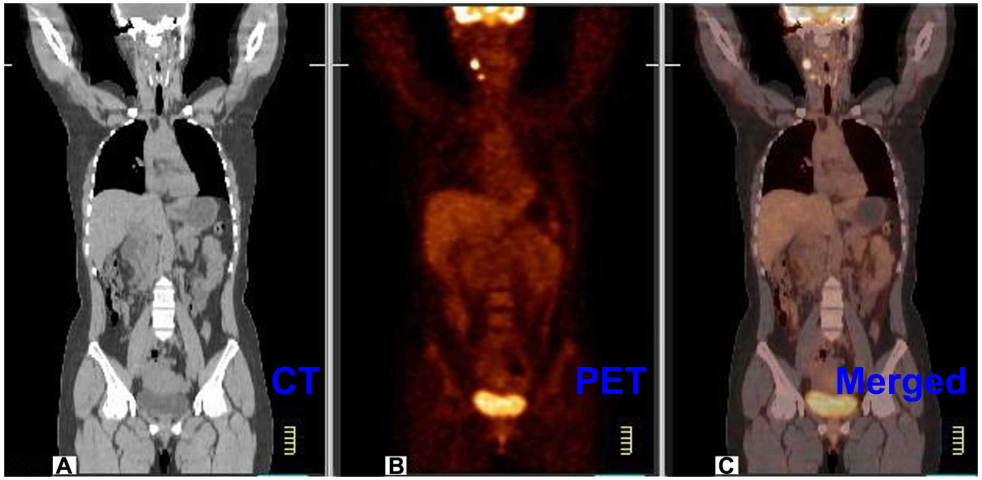
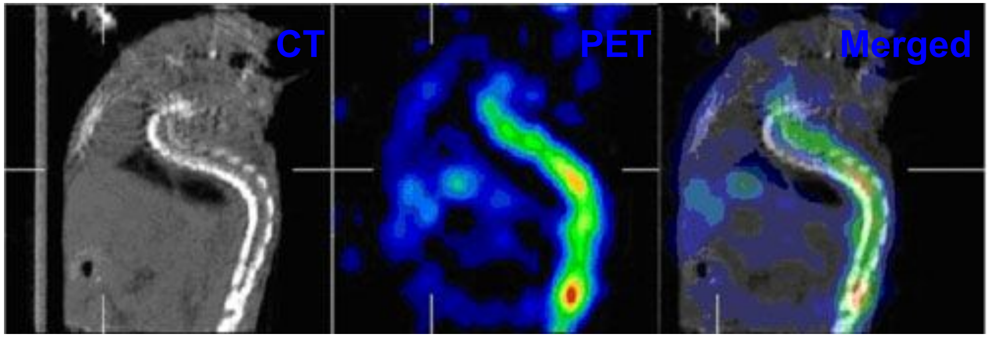


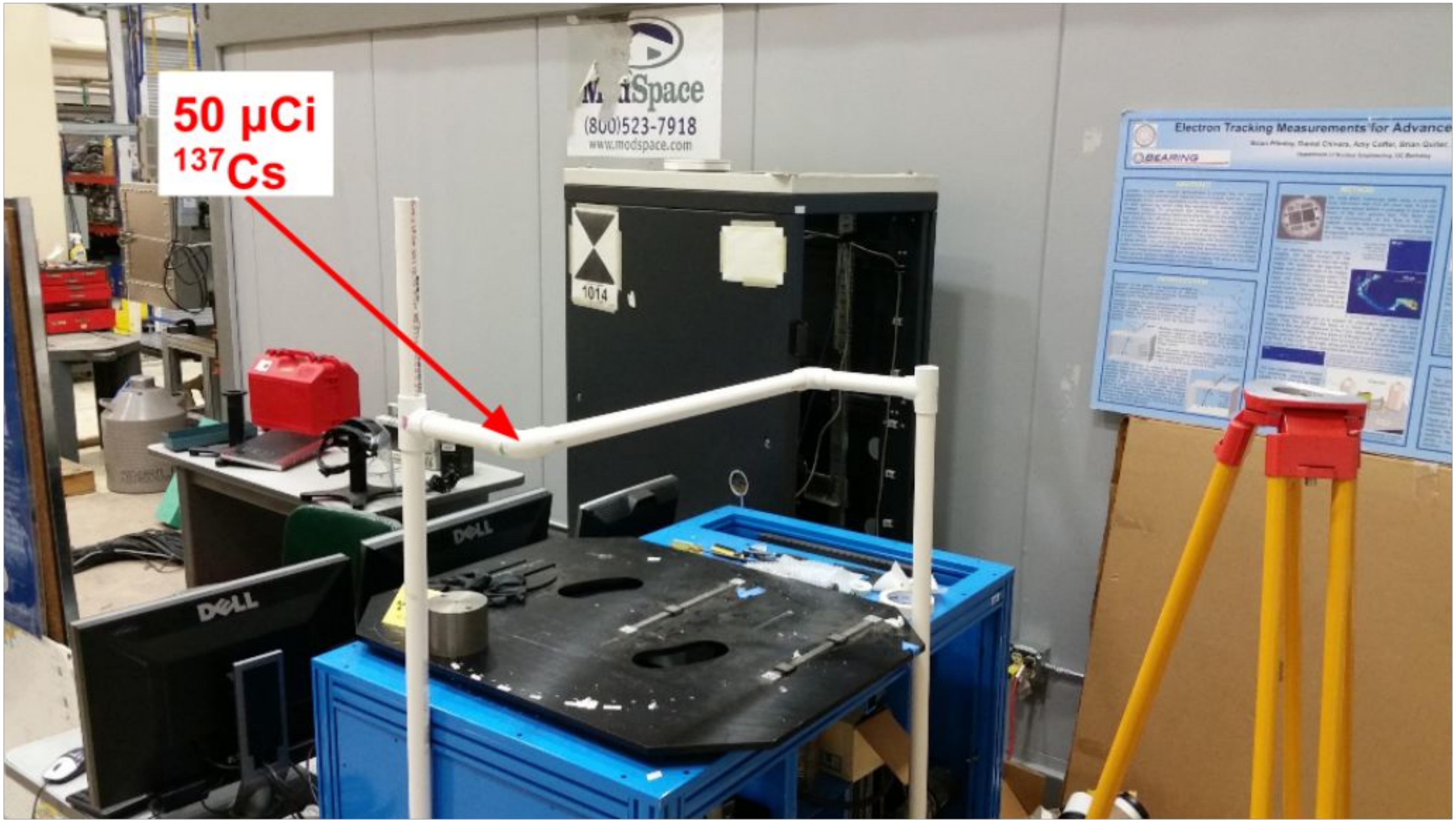
https://www.spandidos-publications.com/10.3892/ol.2016.4229



https://csb.mgh.harvard.edu/mouse_imaging_spect-ct

Image Overlay: Additional Information?

- Merging of images does **not guarantee** additional information
  - E.g. projective ambiguity in 2D image onto 3D model



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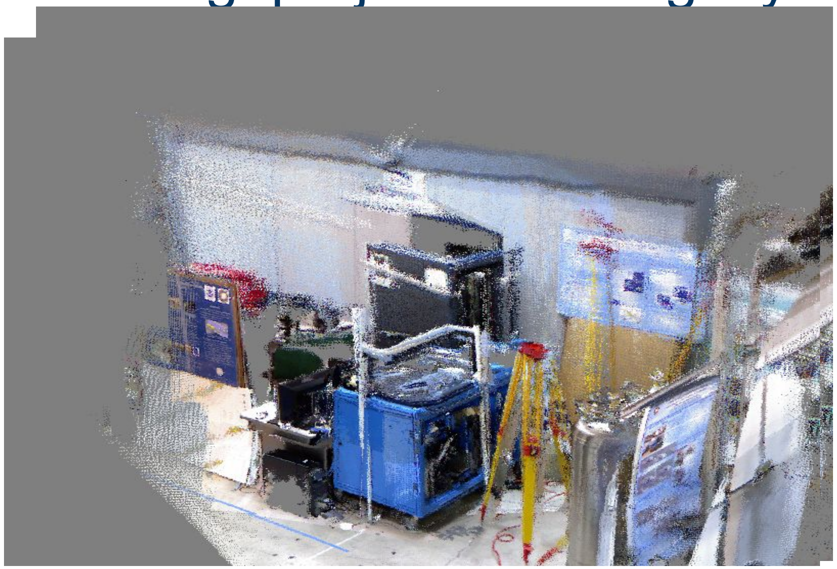
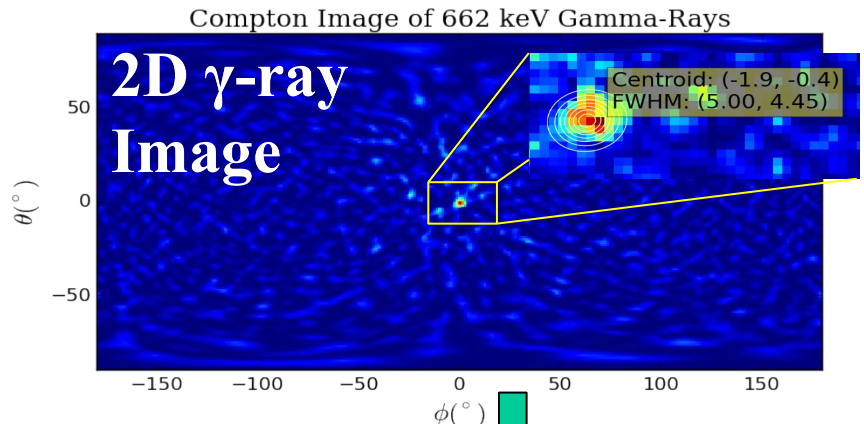
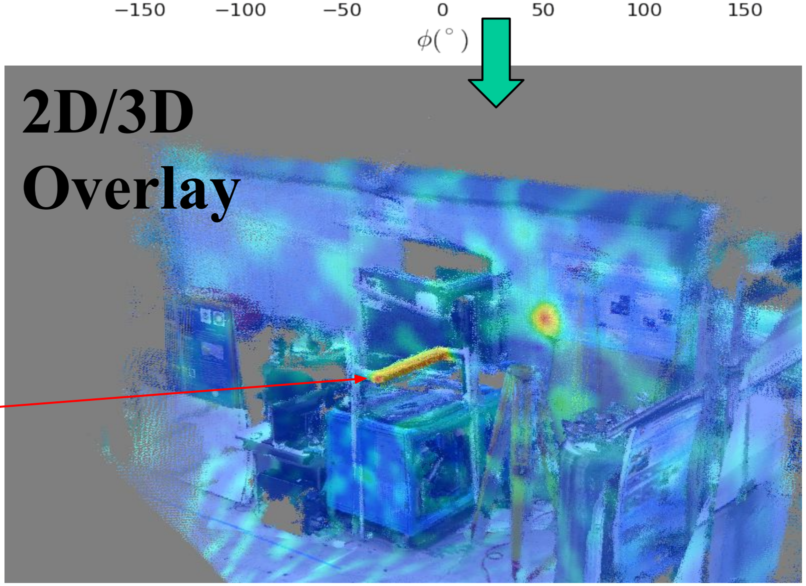






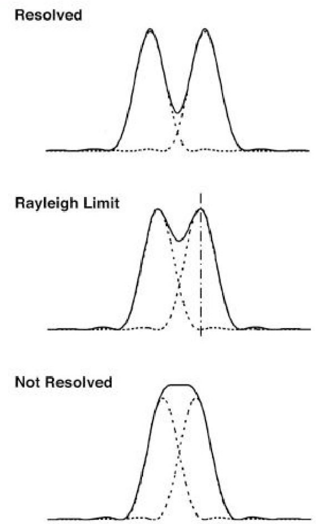
Contrived example, but note ambiguity: 3D overlay does not guarantee depth information!

Image Quality

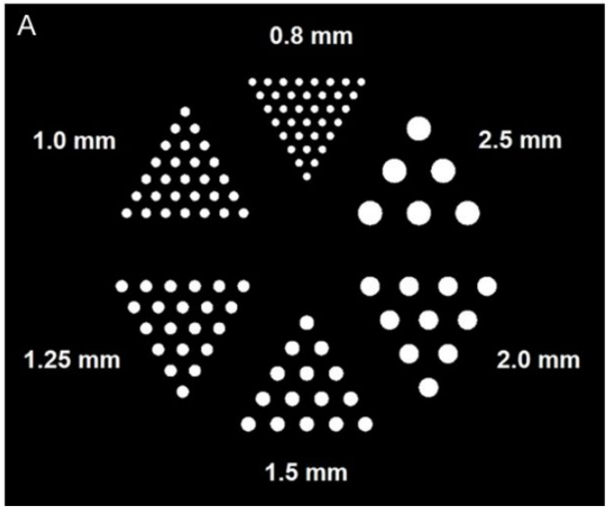
- Evaluate images in terms of three quantities: **resolution, noise (SNR), and contrast (CNR)**
  - Resolution
    - Ability to accurately recover shape, resolve individual features (points, edges, etc)
  - Image noise
    - Quantified in terms of image signal-to-noise ratio (SNR)
  - Image contrast
    - Difference in intensity or “brightness” of features that allows them to be distinguished from one another

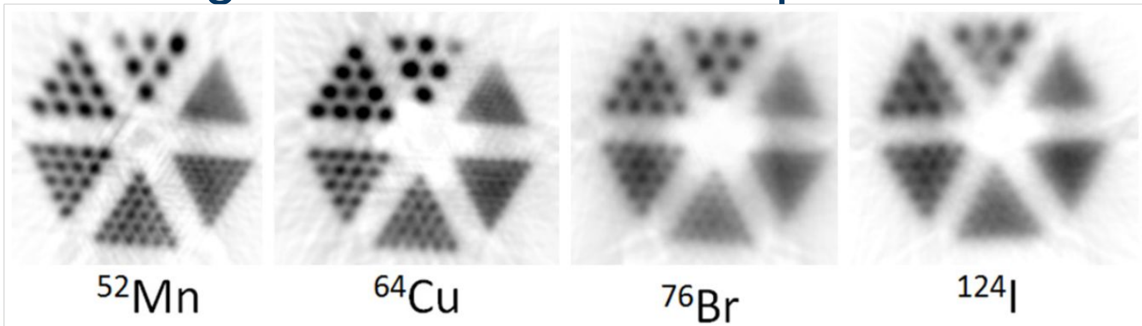
Image Resolution

- Angular (2D) or spatial (3D) resolution
- Rigorous definition of resolution: Rayleigh criterion
    - Ability to discern two points in an image separated by at least one FWHM of the point spread function
- Medical imaging: phantom measurements to quantify resolution
    - E.g. Jaczak or Derenzo phantom



https://pdfs.semanticscholar.org/f838/0f14602091bc51f490a0b14e2bc089abba8a.pdf?_ga=2.224968911.1124053417.1541536030-677040480.1541536030





Derenzo phantom, images from B. Cox et. al 2016

Image Resolution

- Less rigorous (but more common) definition: width of the PSF
    - E.g. Angular Resolution Metric (ARM) for Compton Imaging

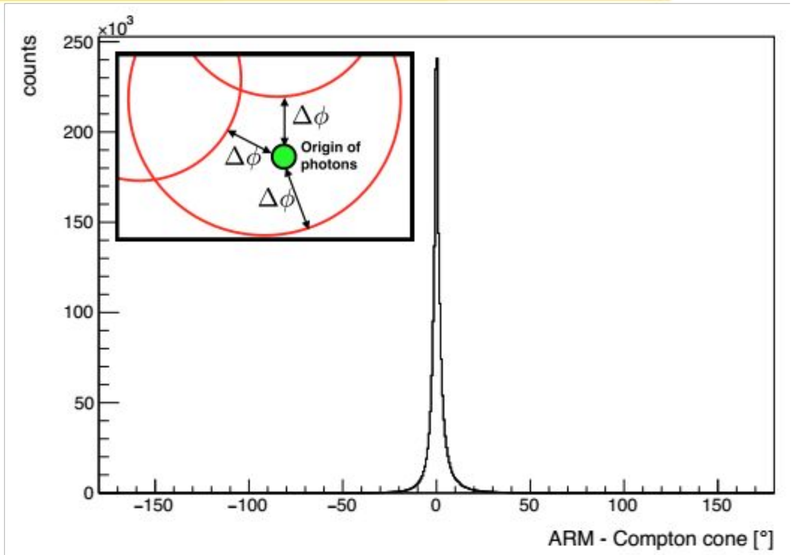
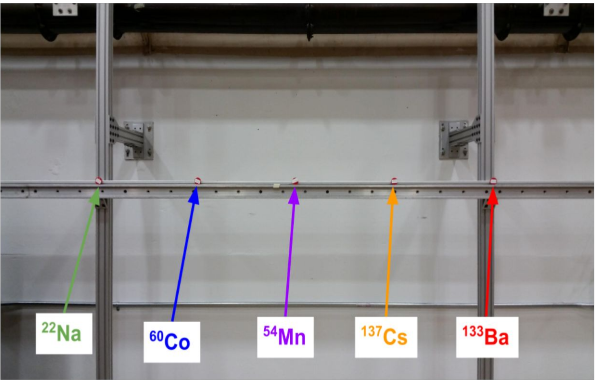
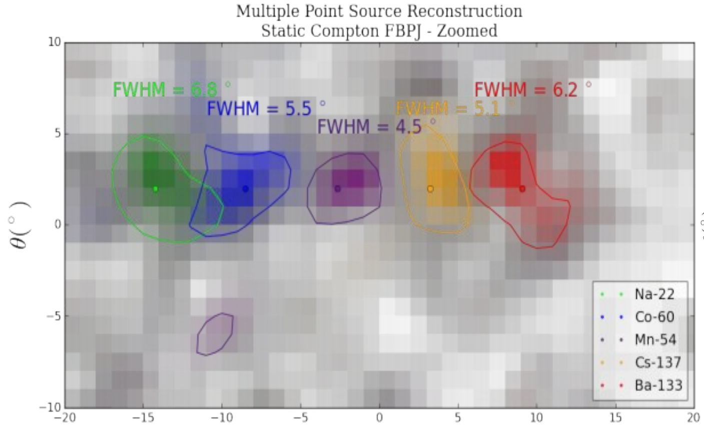
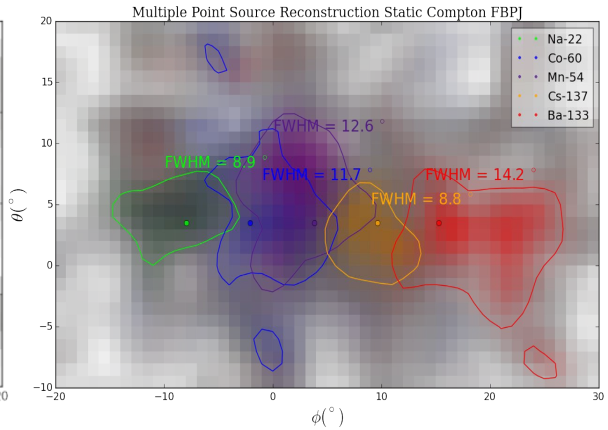


Image from https://arxiv.org/abs/1701.05563



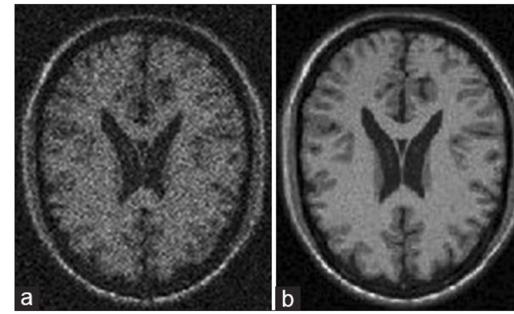




Compton imaging, 2D filtered back projection. From my thesis

Image SNR

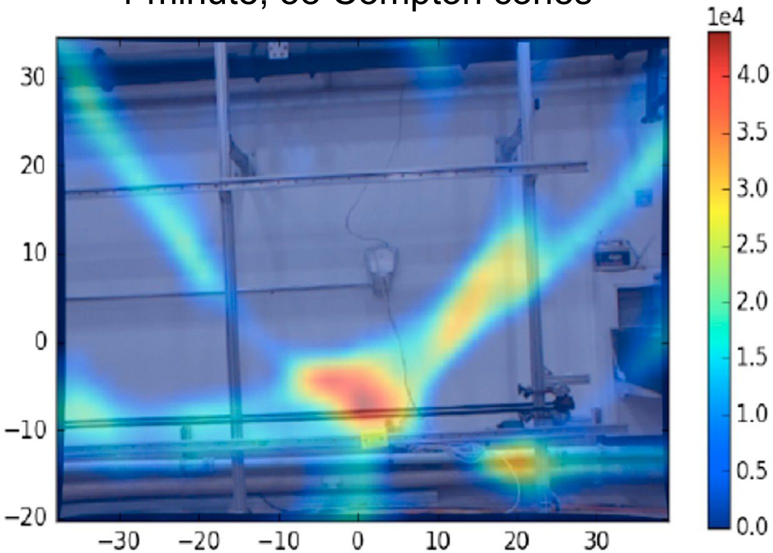
- Ratio of signal to randomness in the image $$\mathrm{SNR} = \frac{\mu_{\mathrm{sig}}}{\sigma_{\mathrm{bg}}}$$
  - Many imaging applications are photon-starved → Poisson noise in image space
- Image noise effects interpretability
  - Contrast-noise ratio (CNR)



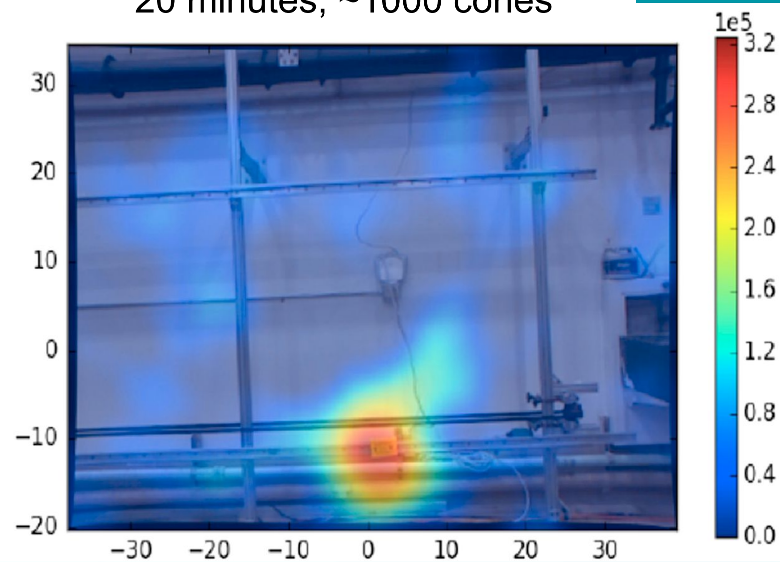
Noise in medical imaging:
a) SNR = 10, b) denoised
From Yadav R. et al 2016

2D Compton image reconstruction via FBPJ taken with HEMI

1 minute, 58 Compton cones

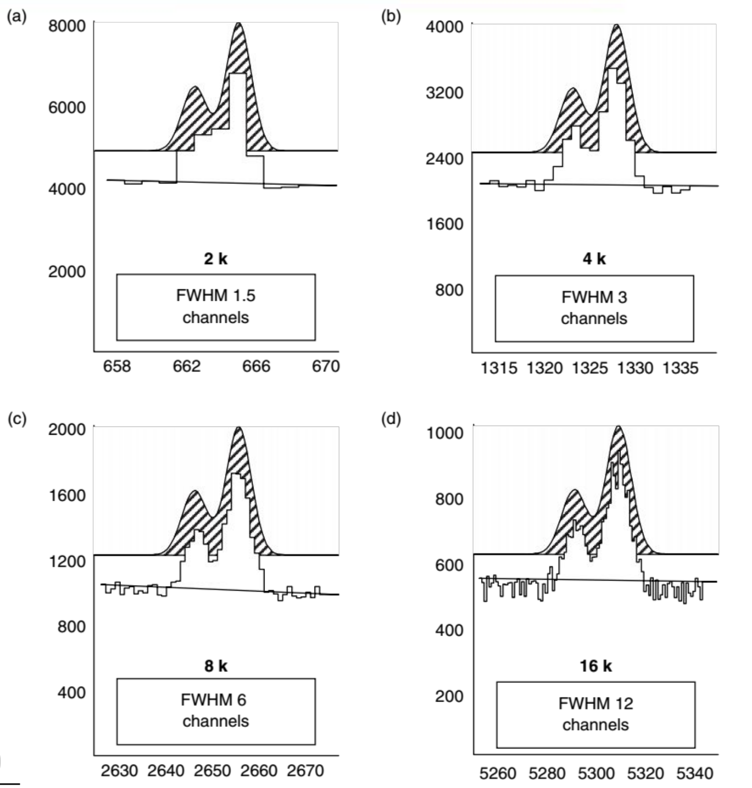


20 minutes, ~1000 cones

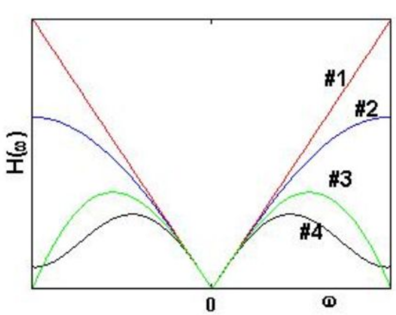


Tradeoff between Resolution and SNR

- Analogy to 1D binning in energy for spectral analysis
  - Fine binning: low quantization error, high Poisson noise
  - Same deal in discretized image spaces!
- Other techniques for controlling the noise/resolution tradeoff
  - Regularization
    - E.g. Tikhonov regularization: $\tilde{F}(\vec{k}) = \frac{|\vec{k}|^2 \tilde{b}(\vec{k})}{1 + \lambda^4 |\vec{k}|^4}$



Gilmore Fig. 4.39

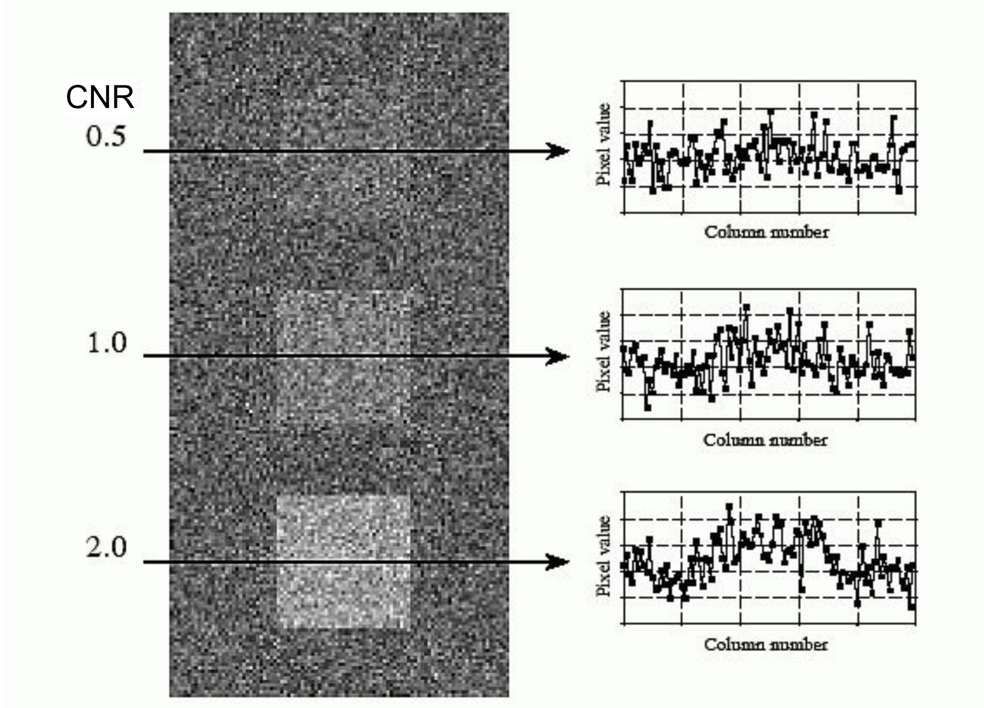


1. Only $|\omega|$
2. sinc filter ("Shepp-Logan")
3. cos filter
4. Hamming filter

Regularized filtered-backprojection. From P. C. Hanaen: Regularization in Tomography

Image Contrast

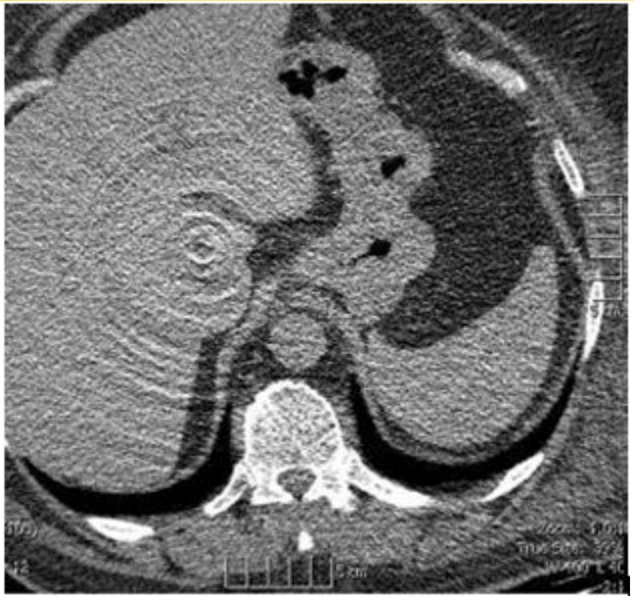
- Intensity (“brightness”) of features in the image
  - Critical in being able to discern/detect features
    - Though not the ONLY consideration
- Normalized by image noise level:
  - $$C = \frac{|S_A - S_B|}{\sigma_o}$$
  - Rose criterion: CNR > 5 guarantees a feature can be identified in the image



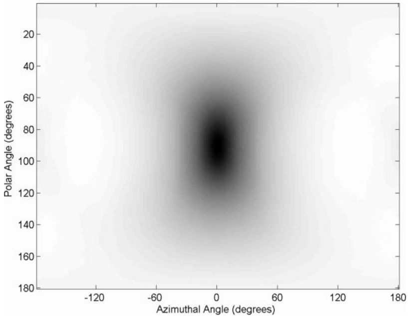
S. Smith: The Scientist’s and Engineer’s Guide to Digital Signal Processing fig. 25-8

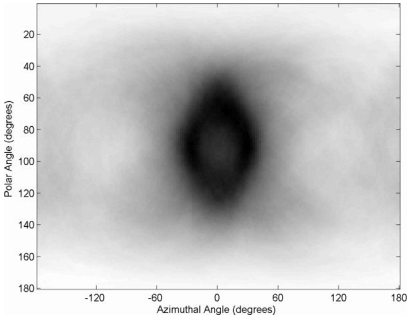
Image Artifacts

- Catchall term for effects that degrade image clarity
  - Image aliasing effects
  - Consequences of reconstruction
  - Merging of projections
- Different than statistical noise!
  - Generally non-random
  - Exhibits coherence in the image space



Ring-artifact in CT image. Univ. of Calgary medical imaging e-book; image credit Dr. Omar Giyab

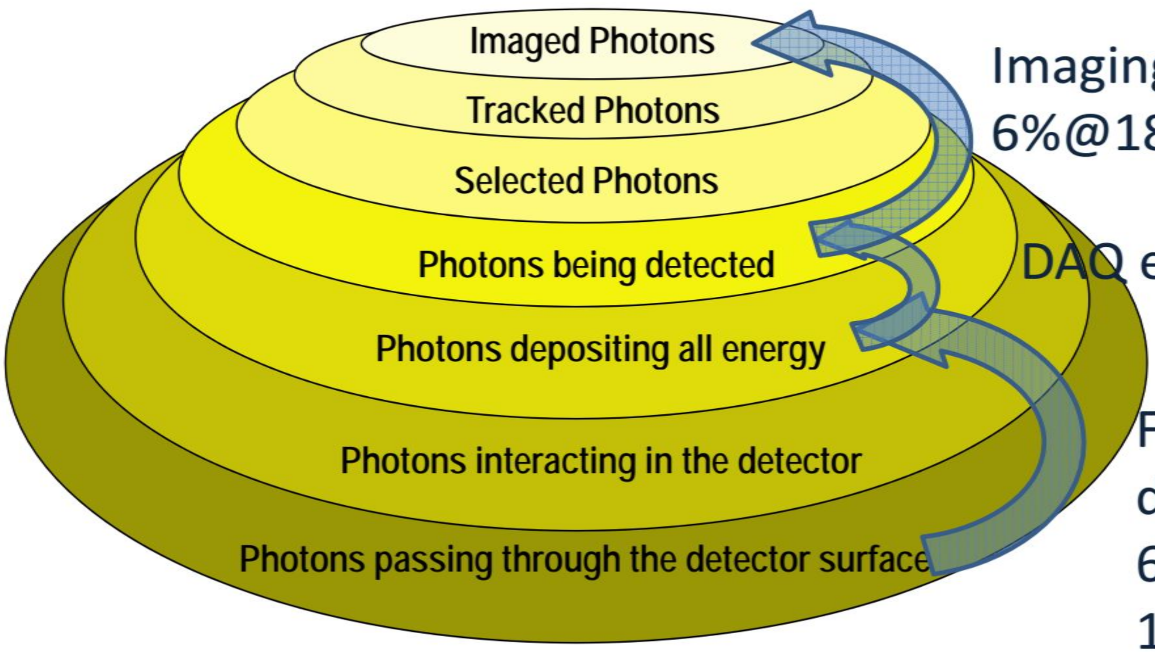




Compton imaging artifacts due to incorrect sequencing of Compton events. Left: correct sequencing, right:incorrect sequencing. From Lehner & He, 2004

Imaging Efficiency

- Inherent tradeoff between **number** of photons and **how much information they carry**
  - Obvious in the case of attenuation-based imaging
  - Even in kinematic imaging, # of photons suitable for imaging is lower than number detected



Cartoon illustrating efficiency loss for HPGe DSSD. From L. Mihailescu, 2009

Image Formation

- Depends on modality
  - Gamma-ray Optics: use lensing effects to focus gamma-rays on a position-sensitive detector
    - Analogous to optical camera, though the “lenses” behave via different physics!
  - Collimated modalities: rely on gamma-ray attenuation to constrain relationship between pixel in detector plane and the image space
  - Kinematic modalities: Use information about the energy & position of gamma-ray interactions to discern the subset of the imaging space from which the gamma-ray could have originated
- N.B. All of these modalities require **position-sensitive** gamma-ray detectors

Image Formation

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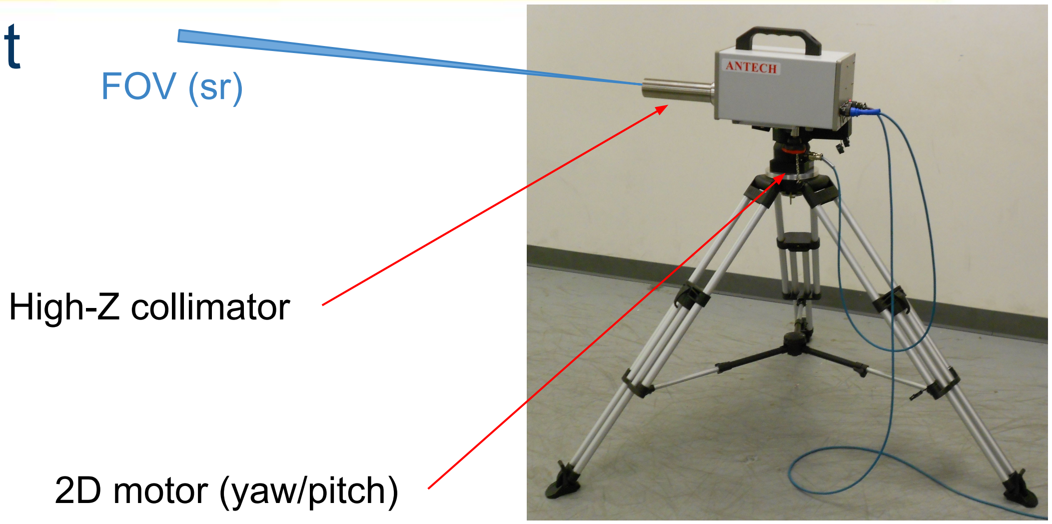
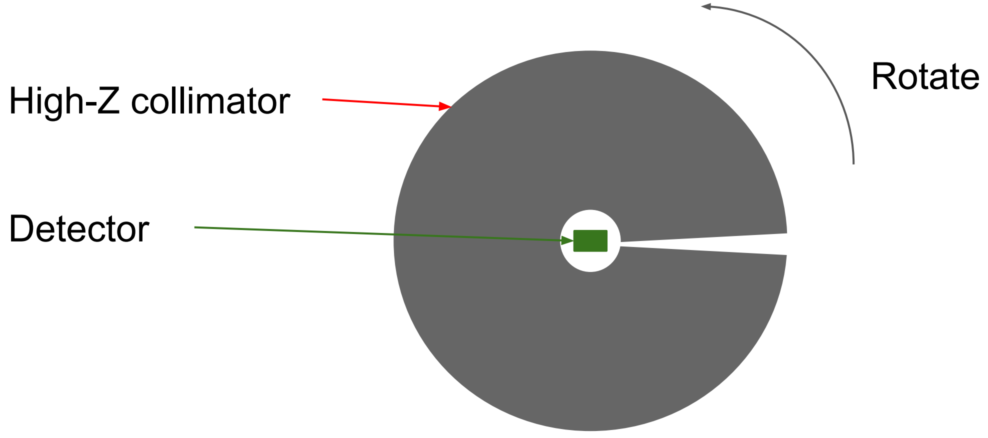
Aside - Imaging with non-position-sensitive detectors

- Collimator to severely limit FOV: countrate from
  - Raster scanner
  - Rotating slit
- Very inefficient
  - Only practical for very high-rate environments

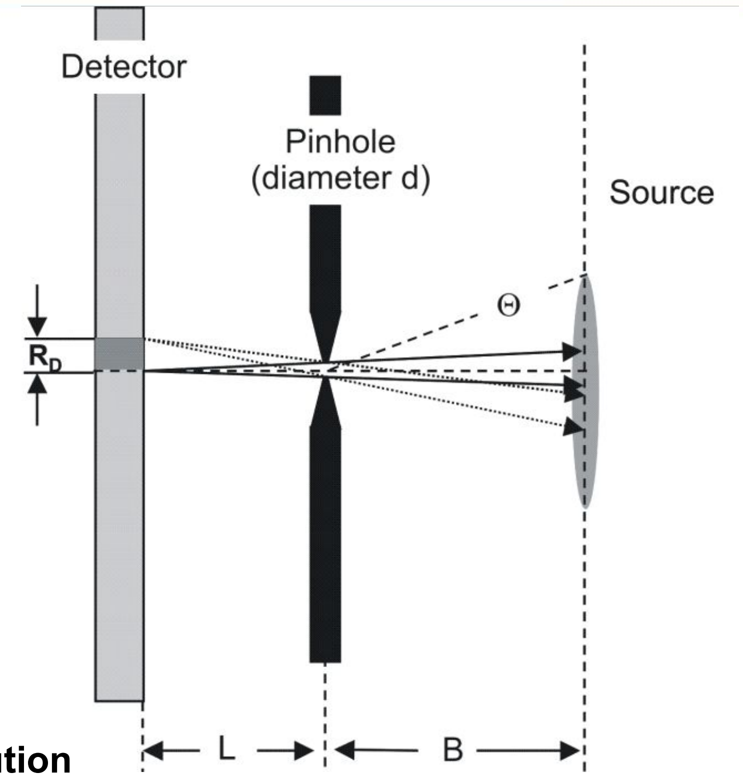


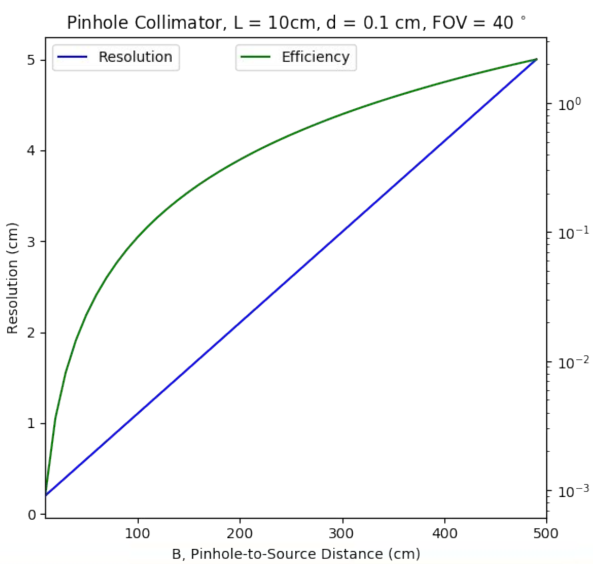
http://www.antech-inc.com/products/g3050/



Collimation I - Pinhole Imaging

- Simple imaging model
- Resolution / efficiency controlled by source-pinhole-detector geometry
  - And detector position resolution





Spatial resolution f/ pinhole

$$R = d + \frac{d}{L} \times B$$

Efficiency

$$\varepsilon < \frac{d^2}{16B^2} \cos^3 \theta$$

Total spatial resolution, including detector position resolution

$$R_{tot} \approx \sqrt{\left(R^2 + \left(\frac{B}{L} \times R_D \right)^2 \right)}$$

Pinhole Imaging

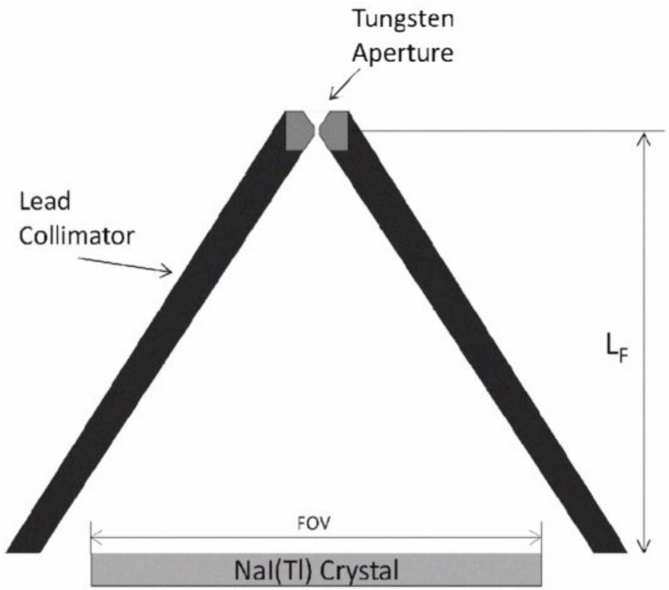


Fig. 1. System configuration [3]

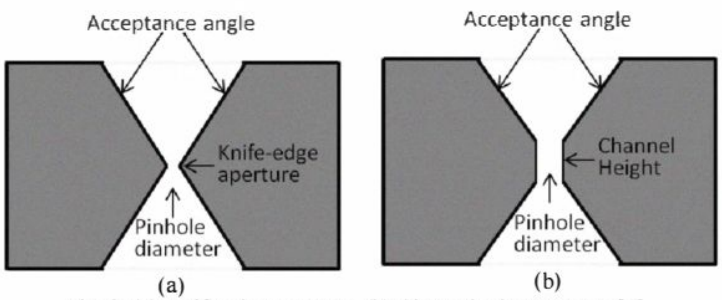


Fig. 2. (a) Knife edge aperture, (b) Channel edge aperture [4]

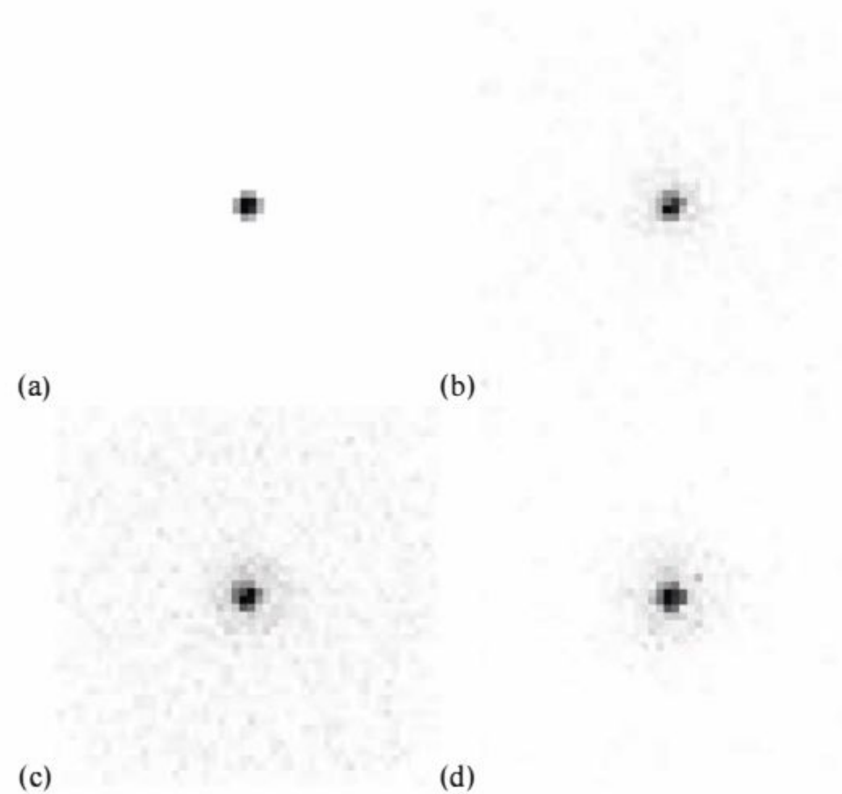


Fig. 7. Projection images of the 0.1 mm point source with 1 mCi activity (a) Tc-99m image with the 15 mm aperture, 1.5 mm pinhole, and 40 ° angle, (b) I-131 at same condition with (a), (c) I-131 image with the 10 mm aperture, 1.5 mm pinhole, and 45 ° angle, (d) 25 mm aperture at same condition with (c)

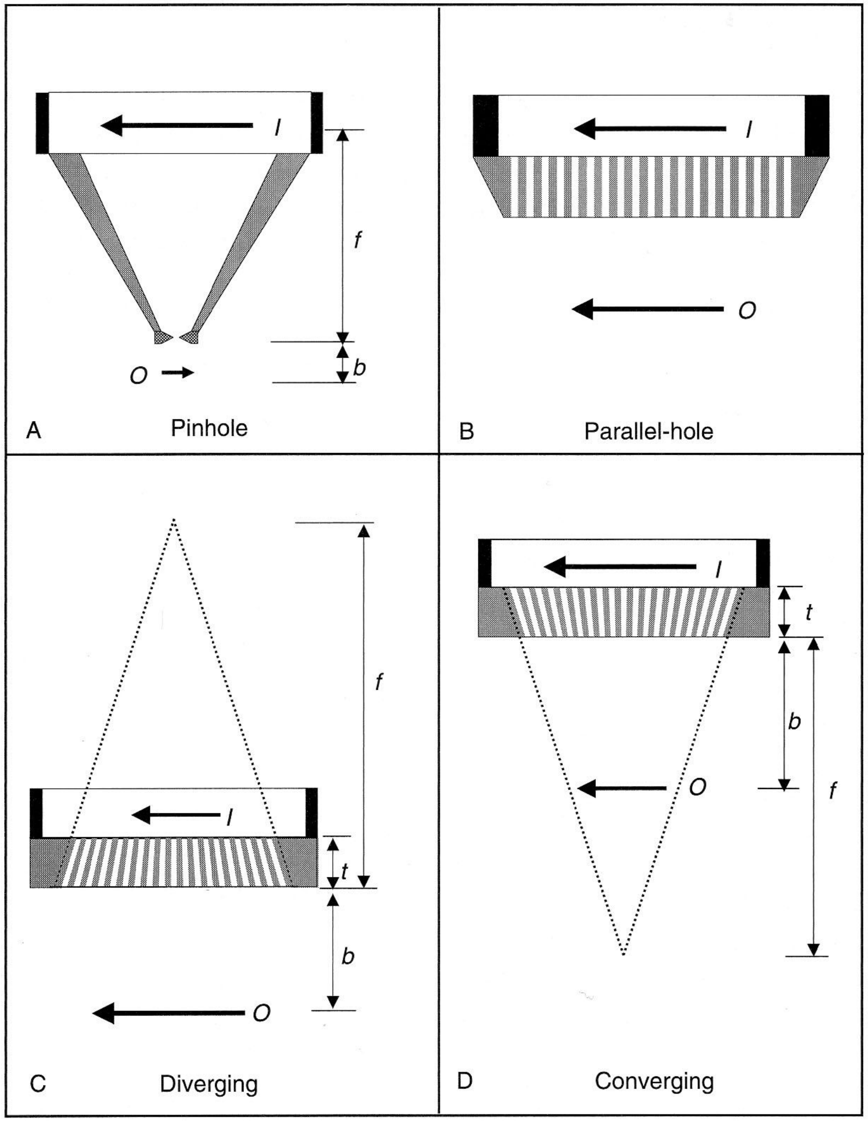
Pinhole designs & example image. From Y.J. Jung et al, 2011

Pinhole Imaging

- Advantages
  - Simple reconstruction
    - Pixel plane = inverted image of gamma-ray source distribution
  - Imaging performance controllable by geometry
    - Can optimize system to get desired performance for constrained imaging
    - Arbitrarily good angular/spatial resolution
- Disadvantages
  - Very inefficient
    - Depends on pinhole & measurement geometry, but typically $< 10^{-6}$
    - Note: this can be an advantage in high-rate scenarios
  - Collimators are bulky & heavy
    - One collimator not necessarily optimal for all applications
  - Limited FOV
    - Again, depends on pinhole & measurement geometry

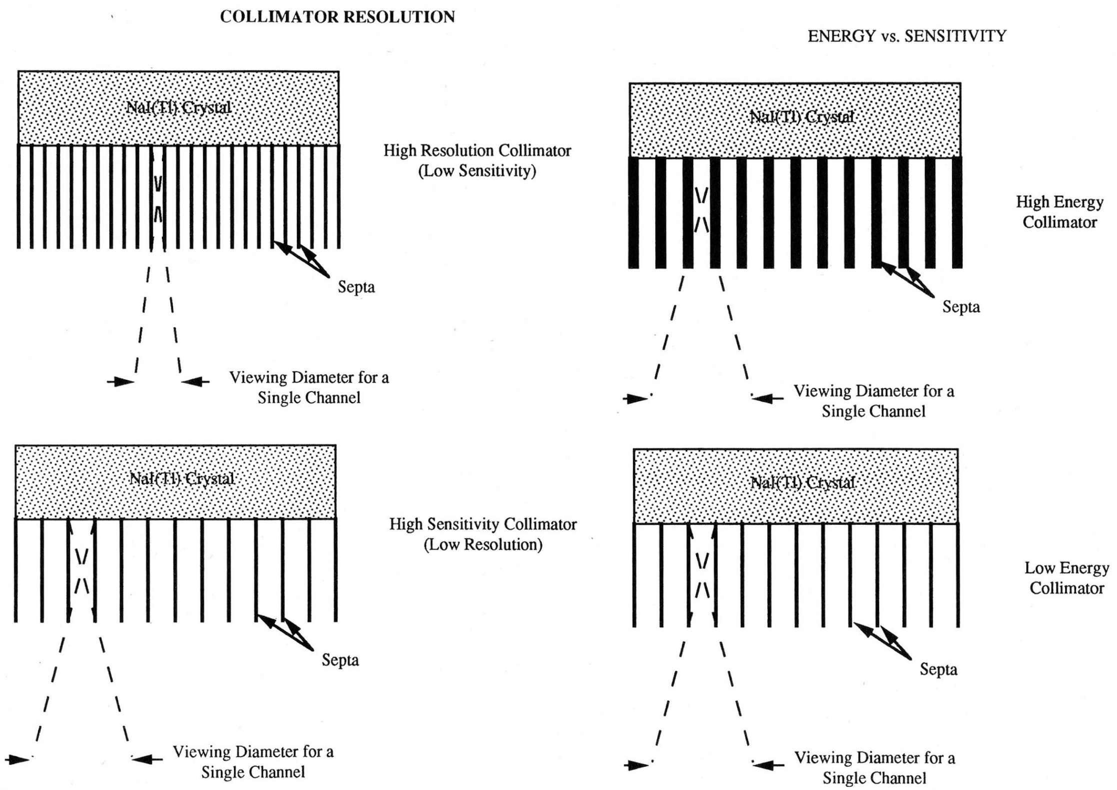
Multiple-Hole Collimators

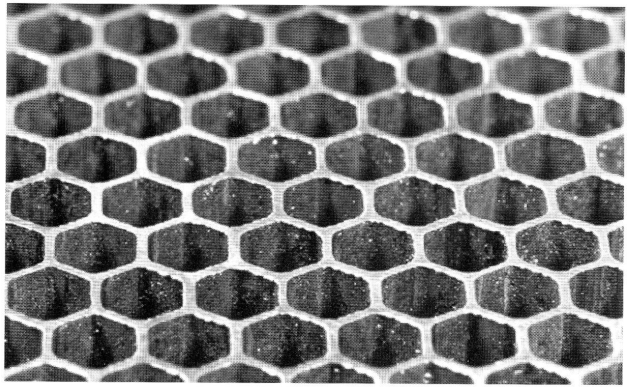
- Different image-formation principle than pinhole
- Increased efficiency compared to pinhole aperture
- Different aperture orientations
  - Parallel-hole
  - Converging/Diverging
  - Dictates FOV, magnification/minification



Parallel Hole Collimator

- Hexagonal or circular apertures most common
- Tradeoffs between image resolution & efficiency, energy & sensitivity

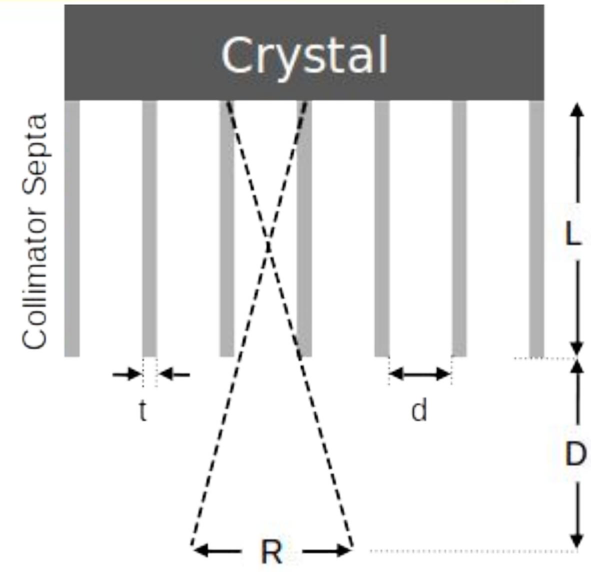




Images from K. Vetter, NE 204 2013

Parallel Hole Collimator Resolution

- Dictated by geometry; depends on:
  - Aperture width (d)
  - Distance to source (D)
  - Collimator length (L)



$$\frac{R}{d} = \frac{D + L/2}{L/2}$$

$$\Rightarrow R = d \times \frac{L + 2D}{L} = d \times \left(1 + \frac{2D}{L}\right)$$

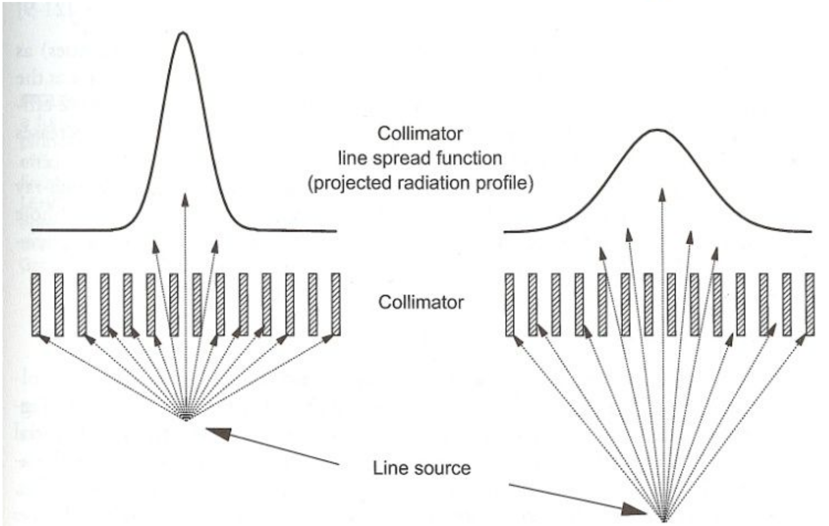
Examples :

Low Energy High Resolution (LEHR) :

$$R = 0.18\text{cm} \times \frac{4\text{cm} + 2(12\text{cm})}{4\text{cm}} = 1.26\text{cm}$$

Low Energy High Sensitivity (LEHS) :

$$R = 0.34\text{cm} \times \frac{4\text{cm} + 2(12\text{cm})}{4\text{cm}} = 2.38\text{cm}$$



Images from K. Vetter, NE 204 2013

Parallel Hole Collimator Efficiency

- Collimator efficiency: fraction of gamma-rays incident on collimator that pass through
  - Doesn't account for detector efficiency!
- Tradeoffs
  - Between resolution and efficiency
    - E.g. length of collimator
  - Between collimator penetration and efficiency
    - E.g. Septum thickness

$$\varepsilon \approx K^2 \left(\frac{d}{L}\right)^2 \left[\frac{d^2}{(d+t)^2}\right]$$

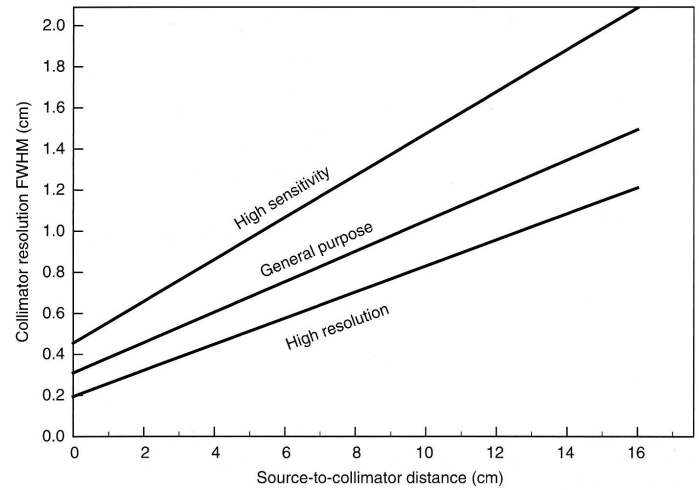
K = aperture shape factor:
~.24 for circular aperture
~.26 for hexagonal
~.28 for square aperture

N.B. Lower res. is better! Inverse relationship btwn eff. & res.

$$R \propto \frac{d}{L}$$

$$\varepsilon \propto \left(\frac{d}{L}\right)^2$$

$$\Rightarrow \varepsilon \propto R^2$$

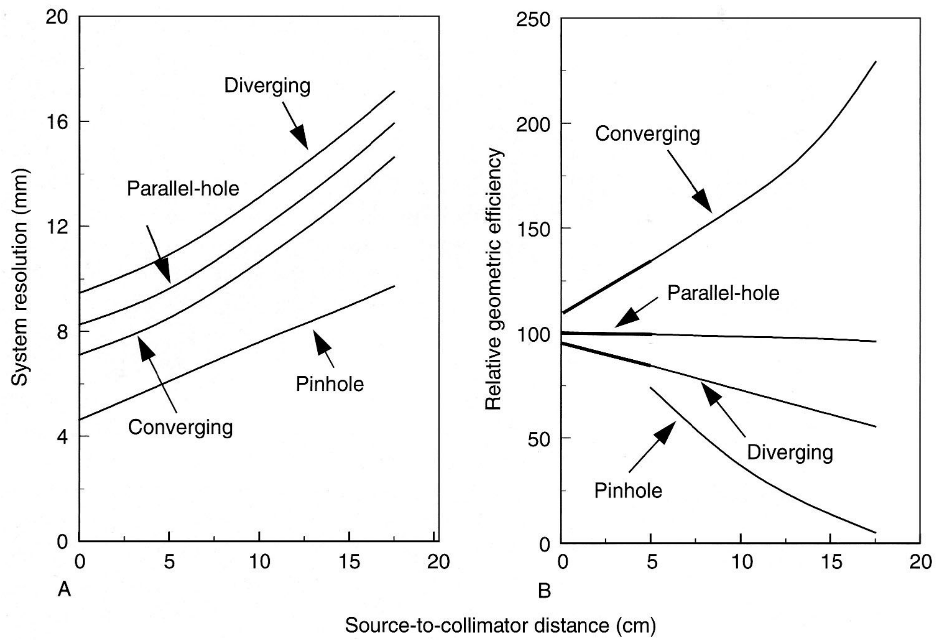


Collimator Type	Recommended Max. Energy (keV)	Efficiency, g	Resolution R_{coll} (FWHM at 10 cm)
Low energy, high resolution	150	1.84×10^{-4}	7.4 mm
Low energy, general purpose	150	2.68×10^{-4}	9.1 mm
Low energy, high sensitivity	150	5.74×10^{-4}	13.2 mm
Medium energy, high sensitivity	400	1.72×10^{-4}	13.4 mm

Adapted from: Hine GJ, Erickson JJ: Advances in scintigraphic instruments, in Hine GJ, Sorenson JA (eds): Instrumentation in Nuclear Medicine (vol 2). New York, Academic Press, 1974.

Multiple Hole Collimators

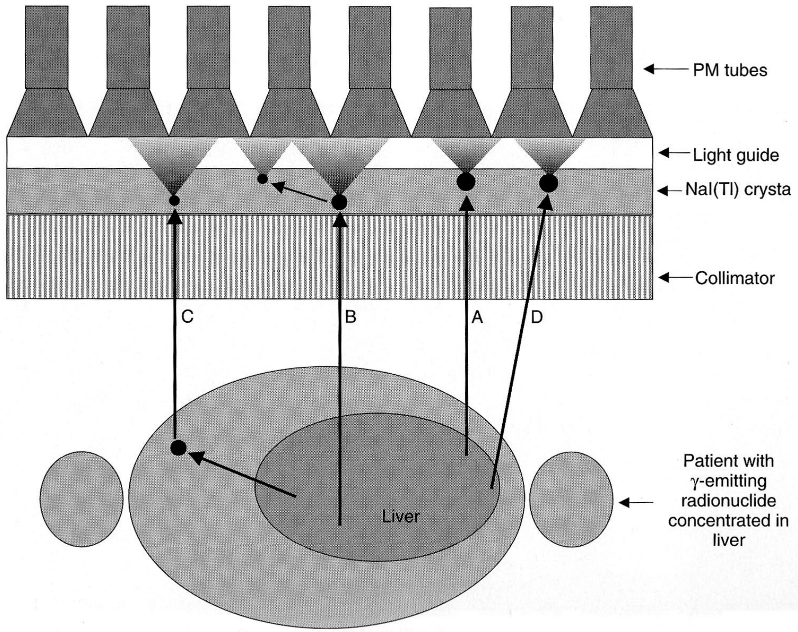
- Magnification:
  - Parallel hole → None
  - Converging → Magnification
  - Diverging → Minification
- Generally higher efficiency than pinhole (~10^{-4} - 10^{-6})
  - Efficiency independent of source-detector distance
- Image resolution controllable by collimator design
  - Spatial resolution depends on source-detector distance

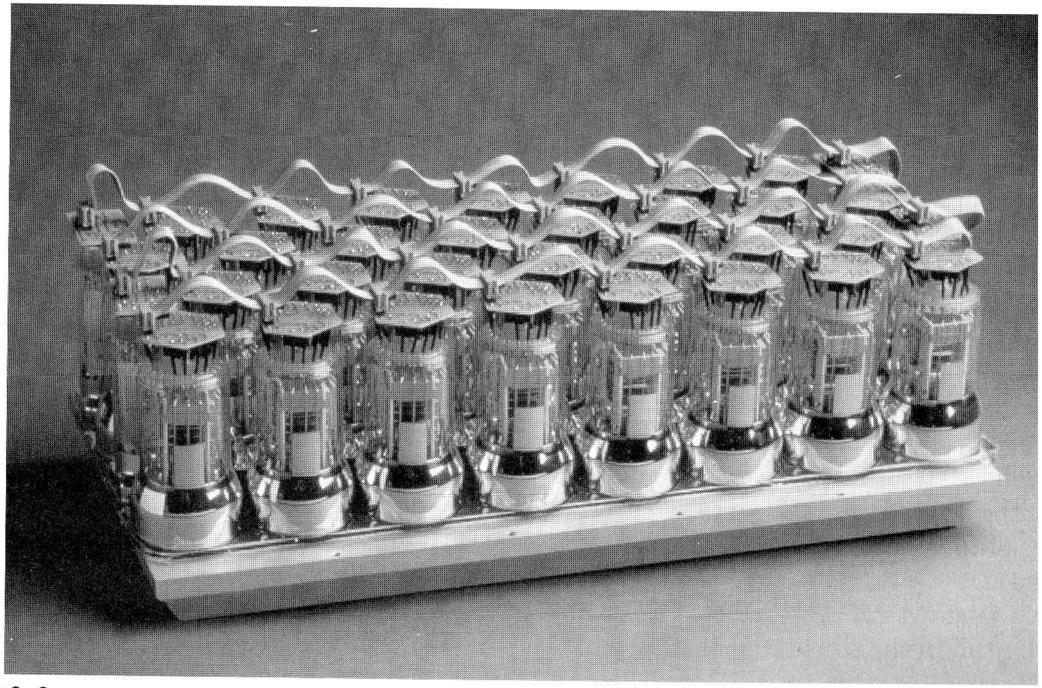


Relative scaling of image efficiency and resolution for different collimators. From K. Vetter, NE204 2013

Position-Sensitive Detectors for Scintigraphy: Gamma-Cameras

- Collimator is only half the battle
- Traditionally, use a scintillation-based “gamma-camera” or Anger camera
  - Invented by Hal Anger @ UCB in the 50’s
  - Thin layer (~1cm) of NaI(Tl) with multiple-PMT readout

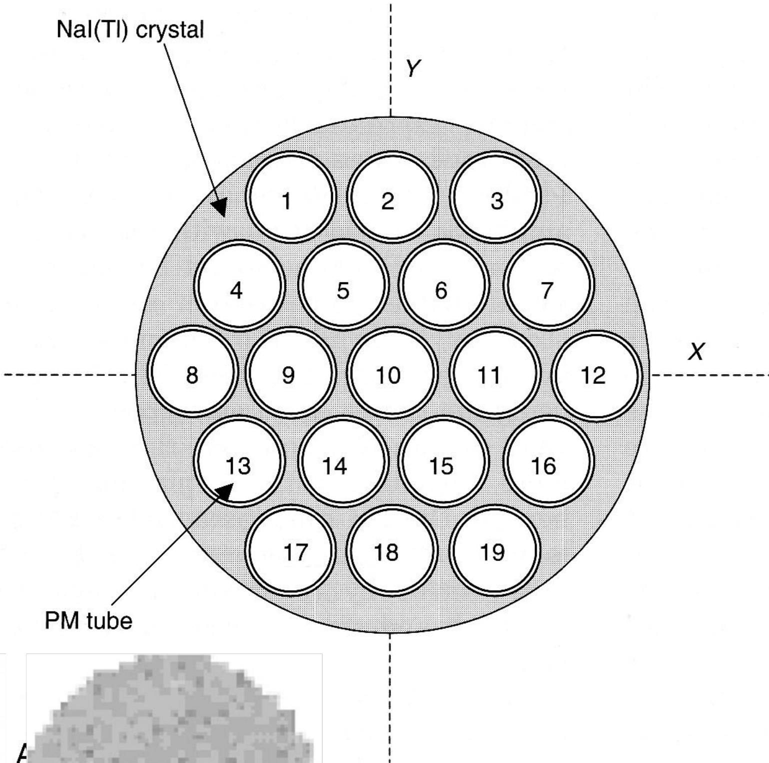




Images from K. Vetter, NE 204 2013

Anger Logic

- 2D lateral (X, Y) position sensitivity based on weighted signal from neighboring PMTs
  - Calibration procedure for position mapping and uniformity correction



$$X = \frac{X^+ - X^-}{X^+ + X^-}$$

$$Y = \frac{Y^+ - Y^-}{Y^+ + Y^-}$$

$$Z = X^+ + X^- + Y^+ + Y^- \quad (Z = \text{total energy})$$

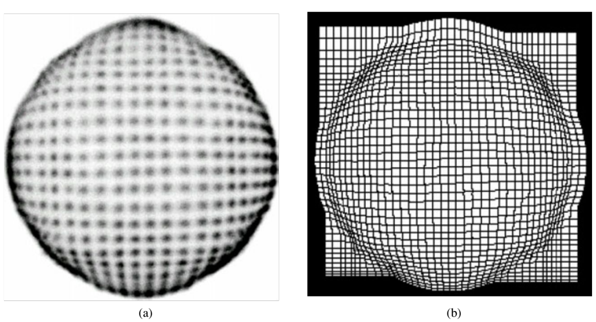


Figure 2. Raw image (a) and position mapped image (b) obtained with NaI(Tl) plate system.

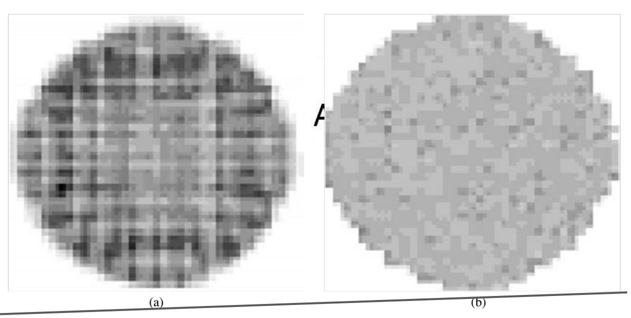
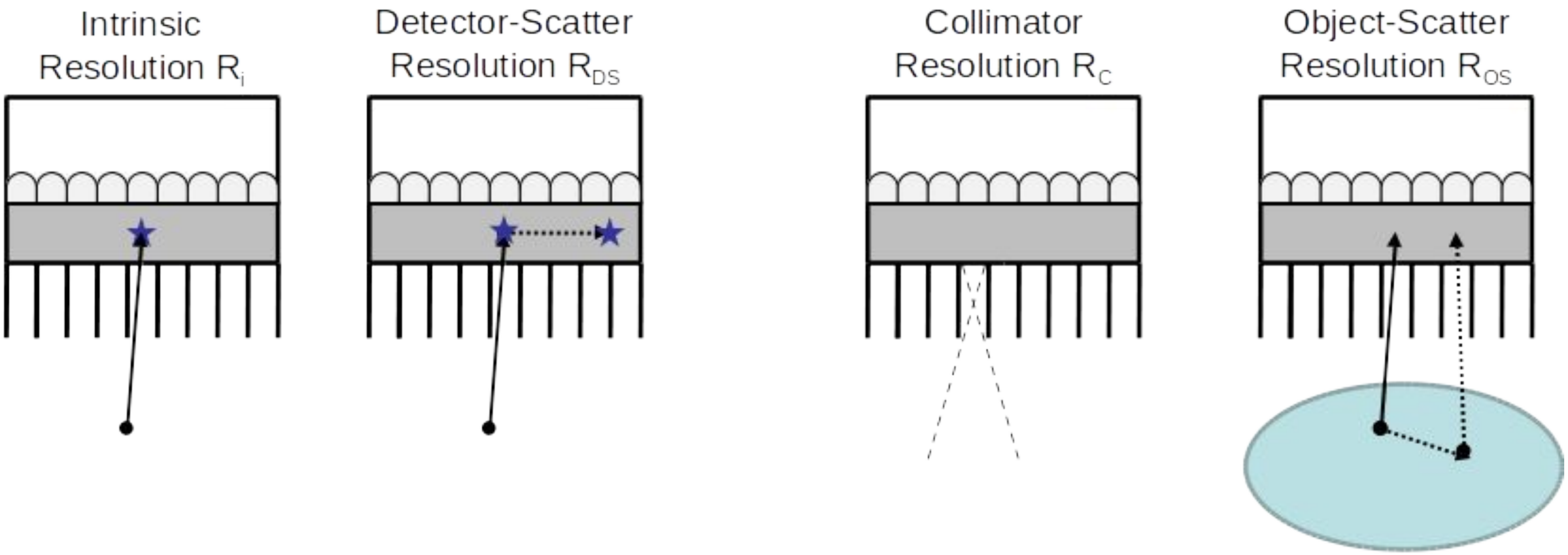


Figure 4. Uniformity correction table image (a), and uniformity corrected image (b).

Images from [M H Jeong et al 2004]

Total Image Resolution of Gamma Camera



System Spatial Resolution

Describes combined effect of all sources of spatial resolution loss:

$$R_{system} = \sqrt{R_i^2 + R_C^2 + R_{OS}^2 + R_{DS}^2}$$

Images from K. Vetter, NE 204 2013

Coded Aperture Imaging

- Pinhole imaging provides good angular resolution, but at the cost of poor efficiency
  - Can we maintain the resolution while increasing efficiency?
    - Yes: Dicke & Ables, 1968 - coded aperture mask for applications in astronomy
  - Must “reconstruct” to deconvolve multi-projection effects

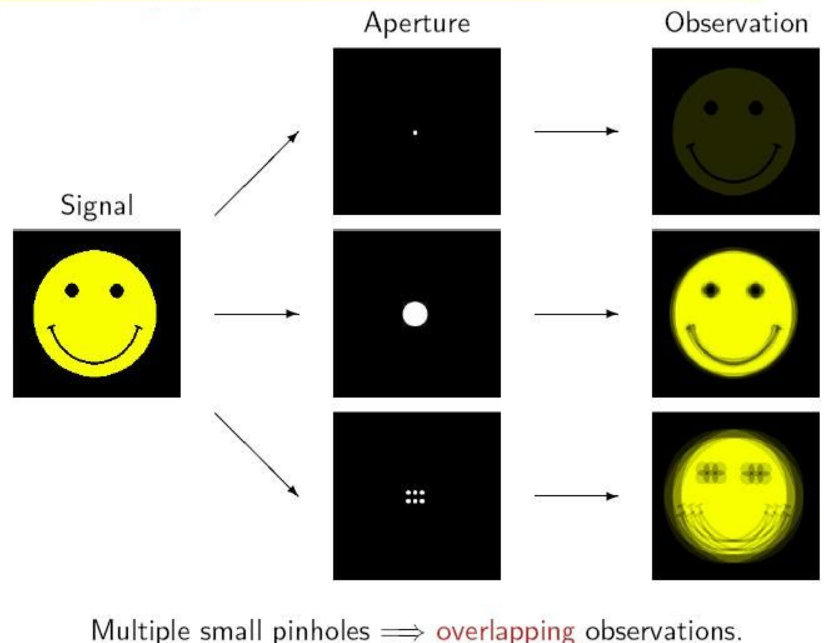
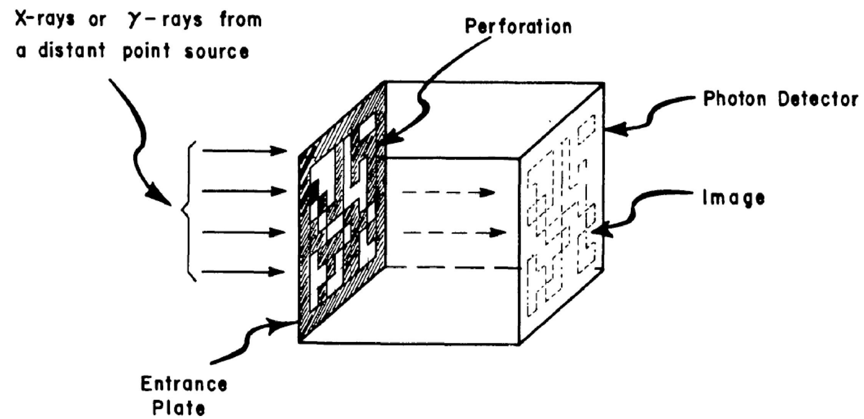


Image from presentation by Marcia & Willet ICASSP 2008



Original conception from Dicke 1968

Coded Aperture Imaging

- Shadowgram encodes unique direction of incident photons
  - Each potential source location/direction in image space should yield unique shadowgram on det. Plane
  - Mathematically: autocorrelation function of mask should be a delta-function on featureless background

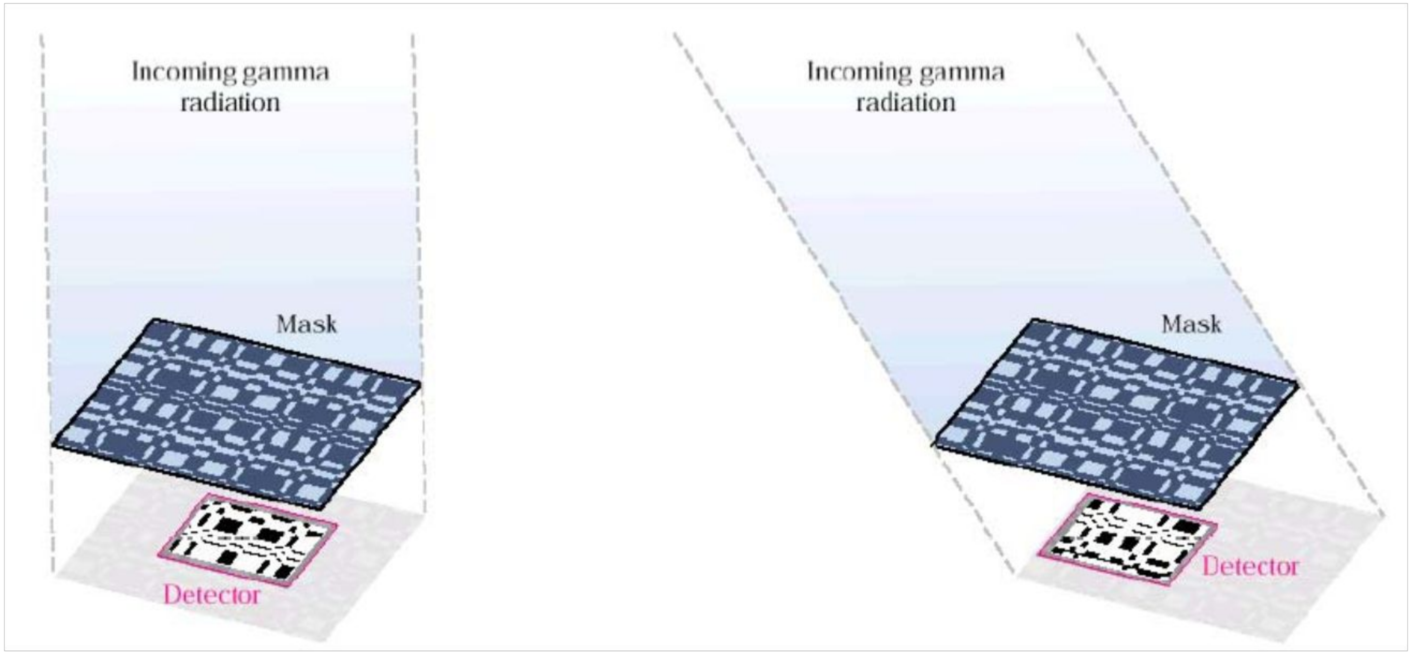
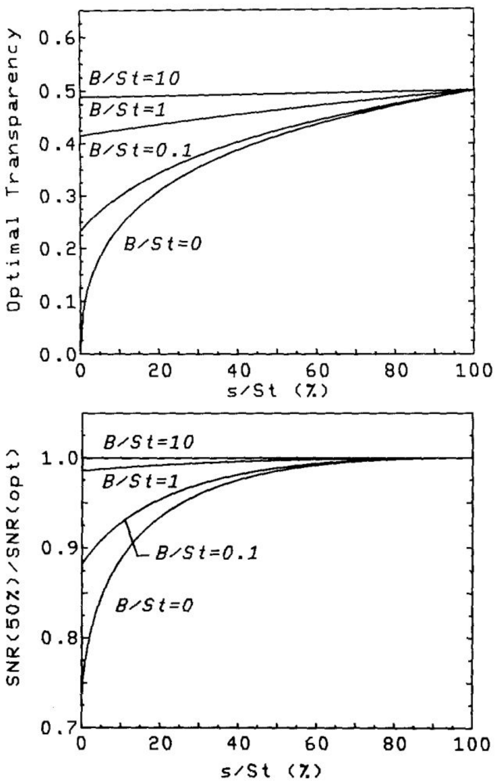


Image from [Berthold Horn presentation on Computational Imaging]

Coded Aperture Mask Design

- Considerations
  - Unique pattern for all relevant directions
  - Uniformity of sensitivity
  - “Opening fraction”
    - Controls degree of coding vs uniform background
- Common designs
  - “Random” apertures
    - Brute-force evaluation of mask pattern
  - Uniformly redundant arrays (URA)



Optimal mask transparency for various source/bgnd conditions. From Caroli 1987

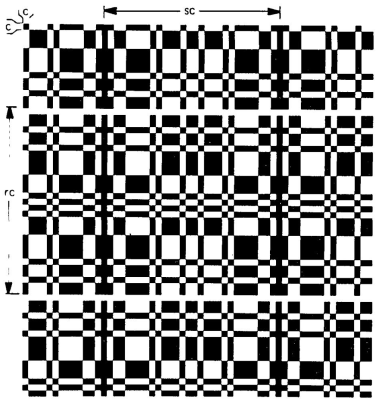


Fig. 1. Two cycles of an $r \times s$ URA pattern. Note it has periods rc and sc with square $c \times c$ pinholes.

Example of an URA. From Fenimore 1980

Decoding the Image

- Different approaches
  - Direct inversion

$$\mathbf{d} = \mathbf{W}\mathbf{s} + \mathbf{b} \longrightarrow$$

$$\hat{\mathbf{s}} = \mathbf{W}^{-1}\mathbf{d} = \mathbf{s} + \mathbf{W}^{-1}\mathbf{b}$$

  - Cross-correlation (pattern matching)

$$I_i = \sum_j M_{i+j} \times D_j$$

$$\text{with}: \; M_{closed} = -1, \;\; M_{open} = 1$$

  - Backprojection
  - Iterative methods
- See Caroli for overview of different “decoding” and reconstruction algorithms

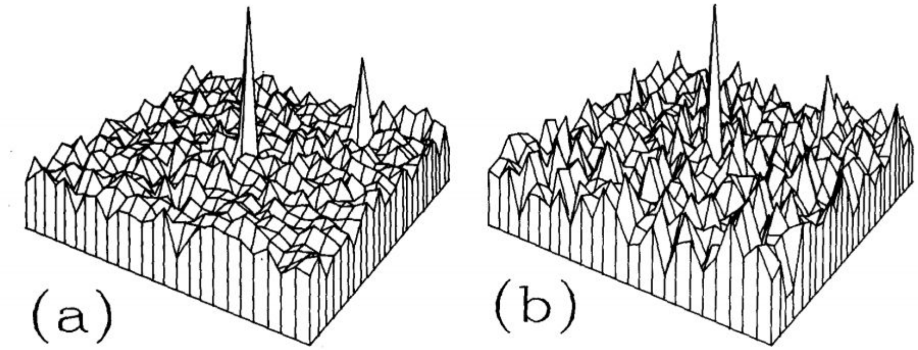
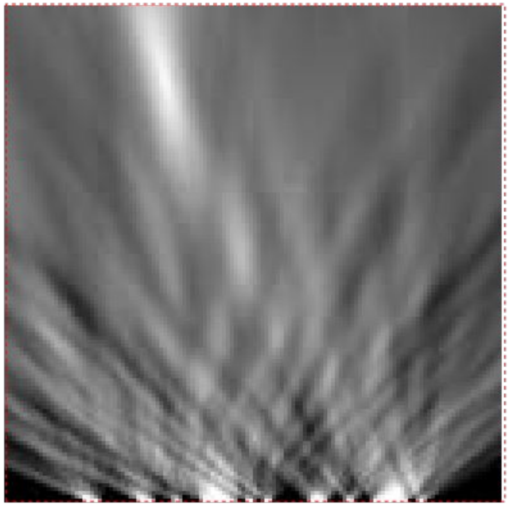


Fig. 2.5. Images of two point sources superimposed on a uniform background, obtained with a random mask (basic pattern 25 × 25) using the following decoding processes: (a) balanced correlation; (b) Matrix inversion (inverse filter). From the picture the image degradation in case (b) is well evident with respect to (a), in particular the lower intensity source almost disappears in the background structure.

Cross correlation vs. direct inversion. From Caroli 1987



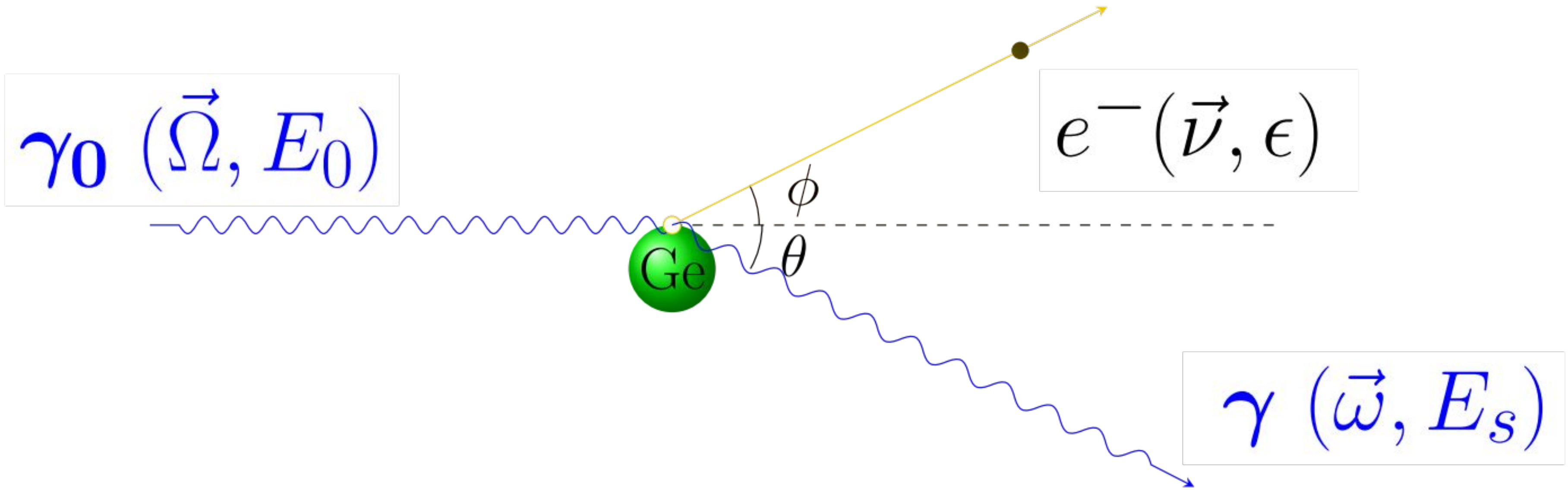
Gamma-ray CA image reconstructed via backprojection from Horn. Original image and animation found here

Coded Aperture Performance

- Image resolution & FOV dictated by source-mask-detector geometry
  - cf. Pinhole camera
  - Systematic effects in partially-coded regions
- Counts from **all** detectors contribute to SNR at **each** image pixel
  - SNR improvements vs. pinhole imaging:
    - High S/B imaging: ~ (# open apertures / # pixels)$^{1/2}$
    - Low S/B imaging: ~(# open apertures)$^{1/2}$
- Artifacts from decoding, near-field effects
  - Magnification of mask pattern on image plane
- Complex (non-point) source distributions?

Compton Imaging

- Imaging based on kinematics of Compton scattering



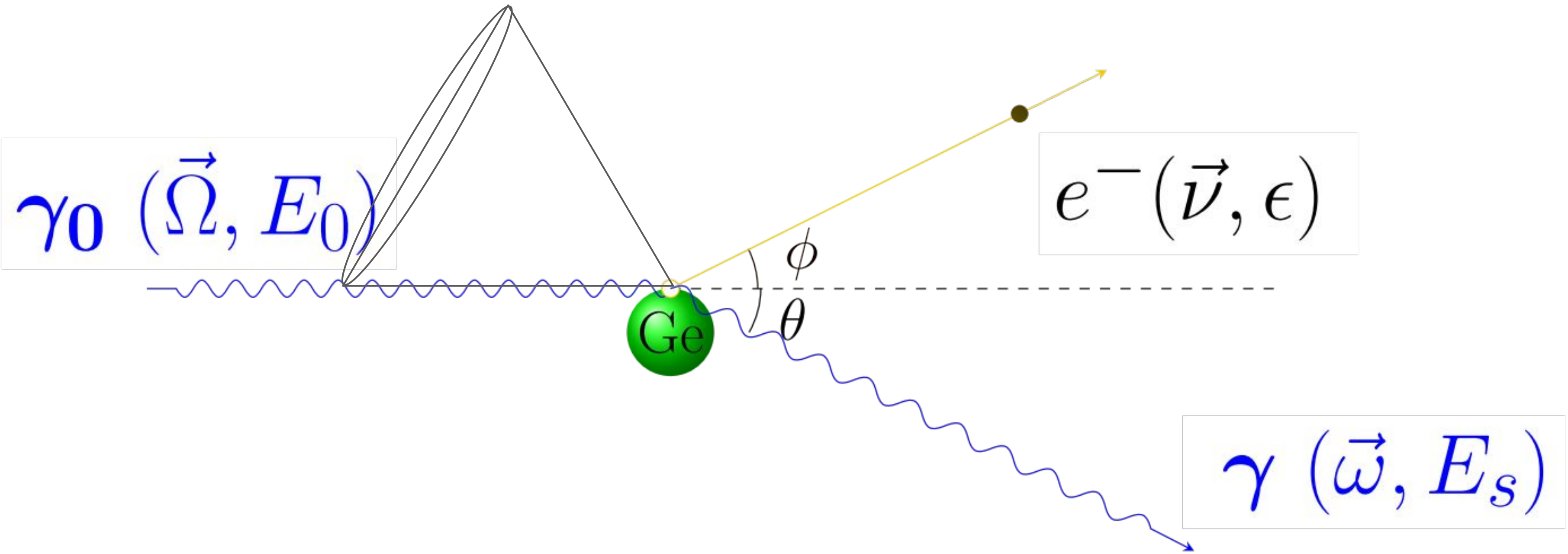
$$\cos\theta = \mu_k = 1 + m_e\left(\frac{1}{E_0} + \frac{1}{E_s}\right)$$

$$\mu_g = \vec{\Omega} \cdot \vec{\omega}$$

$$\mu_k = \mu_g$$

Compton Imaging

- Imaging based on kinematics of Compton scattering



Cone opening angle

Cone axis (scatter direction)

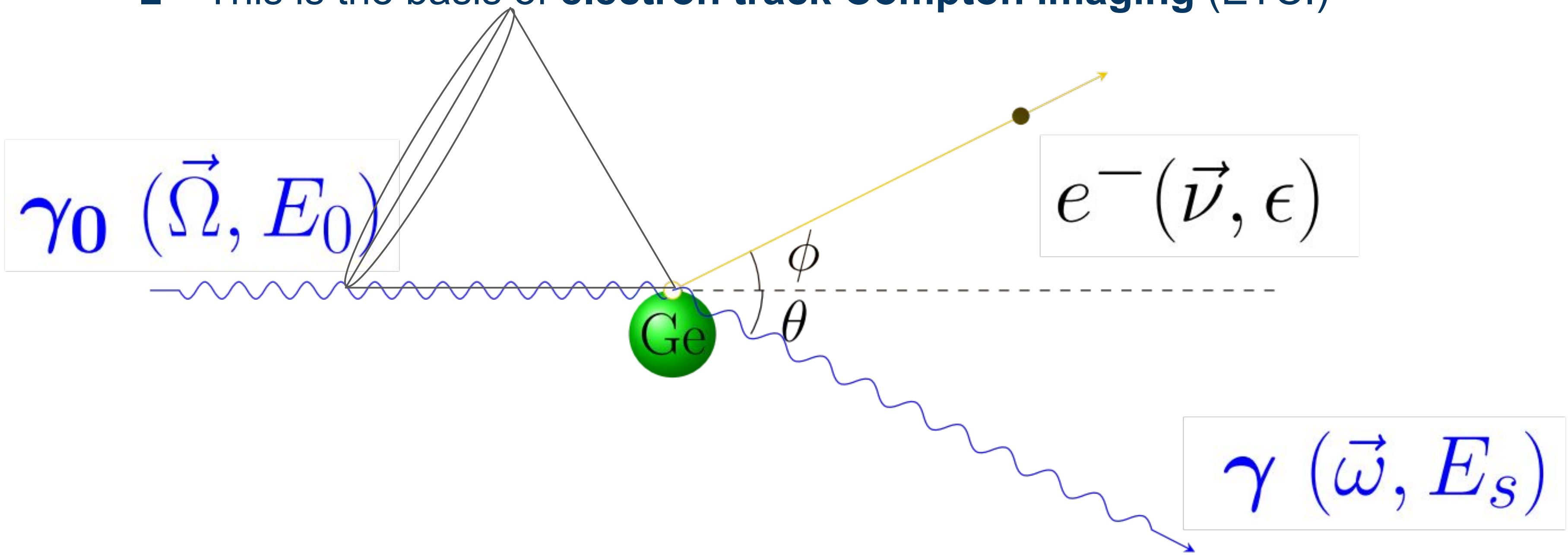
$$\cos\theta = \mu_k = 1 + m_e\left(\frac{1}{E_0} + \frac{1}{E_s}\right)$$

$$\mu_g = \vec{\Omega}\cdot\vec{\omega}$$

$$\mu_k = \mu_g$$

Compton Cones

- Kinematic equation only yields θ
  - Determining ϕ requires information about electron momentum
    - This is the basis of **electron track Compton imaging** (ETCI)



Cone opening angle

$$\cos\theta = \mu_k = 1 + m_e\left(\frac{1}{E_0} + \frac{1}{E_s}\right)$$

Cone axis (scatter direction)

$$\mu_g = \vec{\Omega} \cdot \vec{\omega}$$

$$\mu_k = \mu_g$$

Aside: Electron Track Compton Imaging

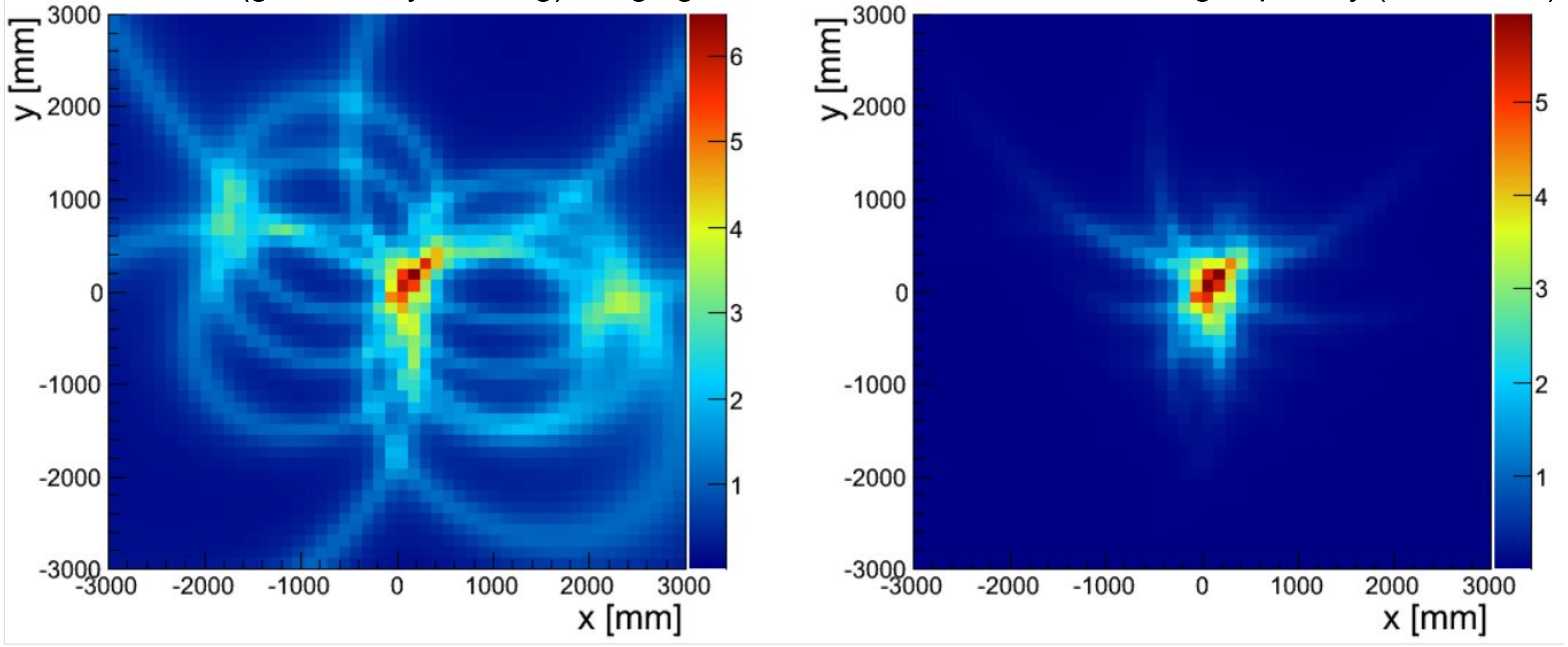
- Knowledge of initial momentum (i.e. scatter direction) of Compton electron constrains ϕ
  - Compton cones → arcs

$$\cos(\phi) = \frac{E_t^2 + E_1^2\left(1 + \frac{2m_oc^2}{E_1}\right) - E_2^2}{2E_tE_1\left(1 + \frac{2m_oc^2}{E_1}\right)^{1/2}}$$

See this paper for further details

Backprojection of Compton cones based on traditional (gamma-ray tracking) imaging

Backprojection of Compton arcs from imager with electron tracking capability (same data)



Images from Mizumura et. al. 2014

Aside: Imaging with *only* Electron Momentum Information?

- Reconstruct **momentum** of incident gamma-ray from electron track information only (no tracking of scattered gamma-ray)
  - Removes coincidence requirement
  - Useful for inefficient electron-tracking devices (gas dets, CCD's)

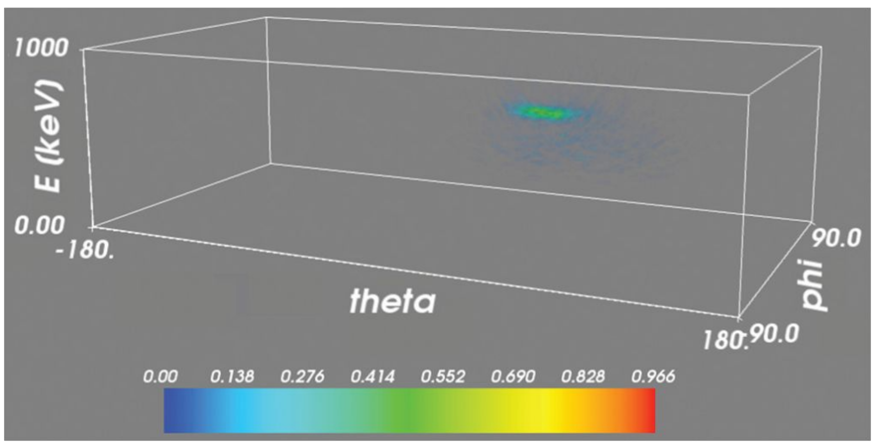


FIG. 4. Showing a 3-D representation of the energy image from a Cs-137 source measurement.

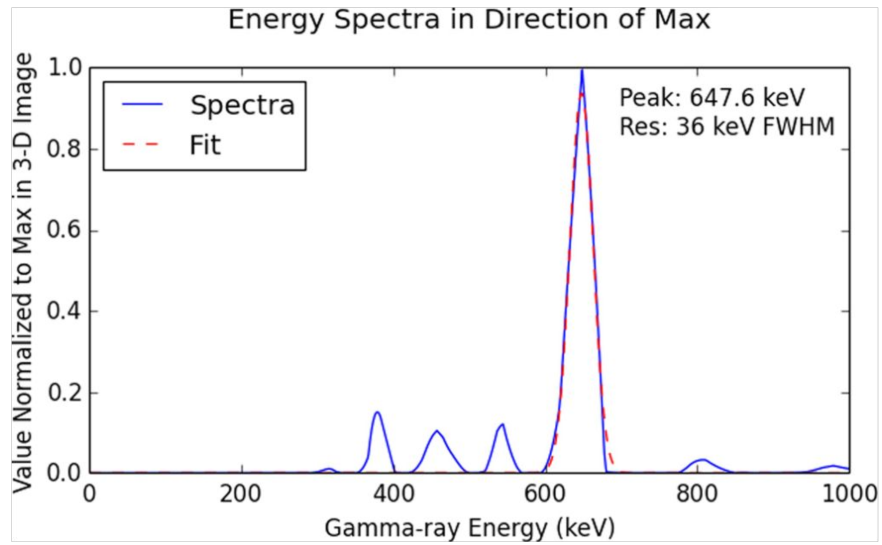


FIG. 6. The spectra through the source maximum value. The 662 keV peak is observed.

Images from Haefner et. al. 2014

- N.B. the image space is 3D: with 2 angular and 1 energy dimension!

Required Information for Traditional Compton Imaging

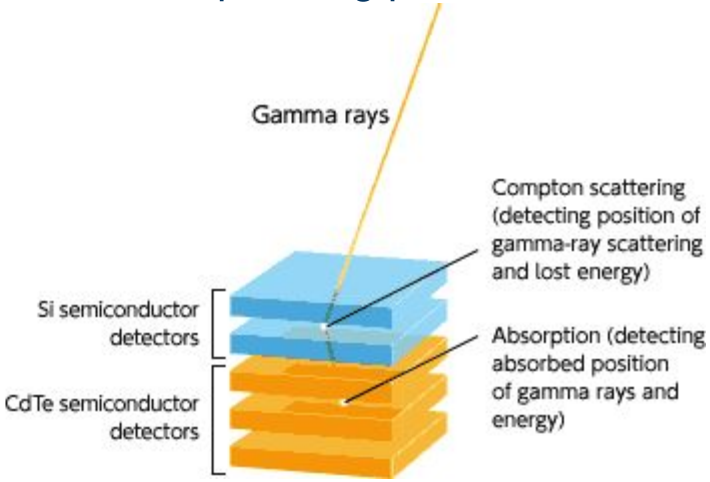
- The Compton cone is defined by two parameters:
  - Cone axis
    - Opposite of scatter direction of gamma-ray
    - Thus need to know the position of the **initial Compton scatter** and the position of a subsequent interaction by the **scattered gamma-ray**
      - Also need to know the order or *sequence* in which these interactions occurred
  - Cone opening angle
    - This is determined from the kinematic equation
    - Need to know the energy deposited by the gamma-ray in the initial Compton scatter interaction
    - Also need to know the energy of the incident gamma-ray
      - Can either determine spectroscopically, or by the sum of energy deposited by coincident interactions

Compton Imaging FOV

- No collimator → Inherently wide FOV
- Systems can be designed with omnidirectional imaging capability, i.e. 4π FOV

Scatterer/Absorber

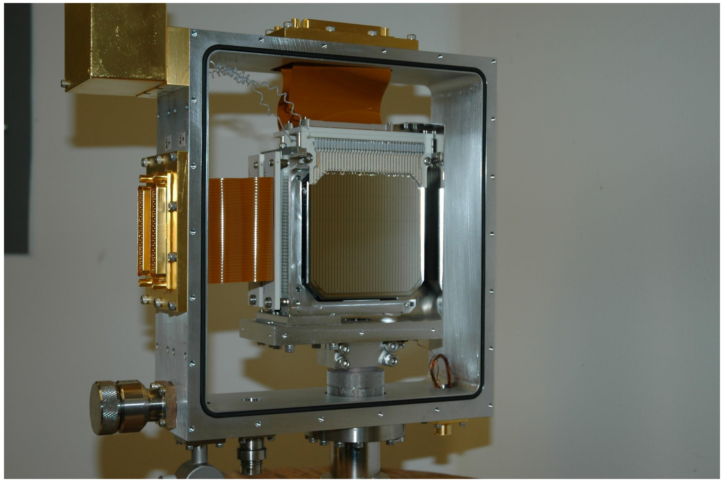
- Non-uniform sensitivity
- Narrow FOV applications like astronomy
- Event sequencing problem constrained



Scatterer/Absorber geometry - Based on Astro-H instrument. Image from [JAXA]

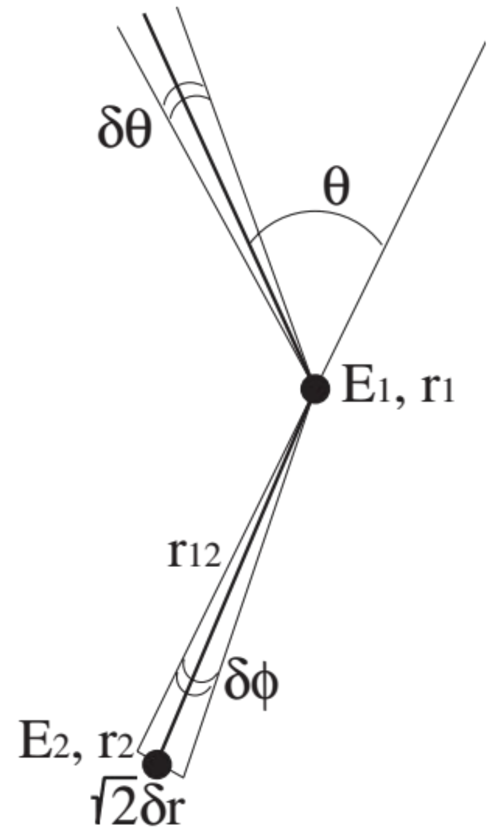
Omnidiretional geometry

- More uniform imaging over 4π
- Useful for many nuclear security applications



Compton Imaging Efficiency & Resolution

- Efficiency depends on many factors - requires significant system characterization
  - Detector material & geometry
  - Event reconstruction
- Image resolution depends on
  - Position resolution of detector
  - Energy resolution of detector
  - Fundamental limits?
    - Compton equation assumptions
    - Doppler broadening: see here



$$\delta\cos\theta = \left(\frac{1}{A_0^4}\delta^2 A_0 + \frac{1}{A_d^4}\delta^2 A_d + 2(1-\cos^2\theta)\frac{\delta^2 r}{r_{12}^2}\right)^{1/2}$$

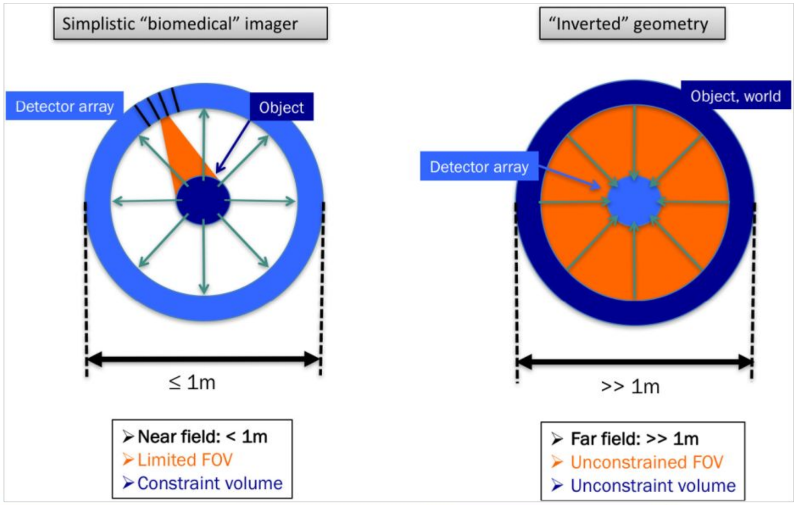
SPEIR: L. Mihailescu et. al. 2007

Compton Image Reconstruction

https://github.com/lbl-anp/GammaRayImagingWorkshop_EBSS2018

Volumetric Imaging

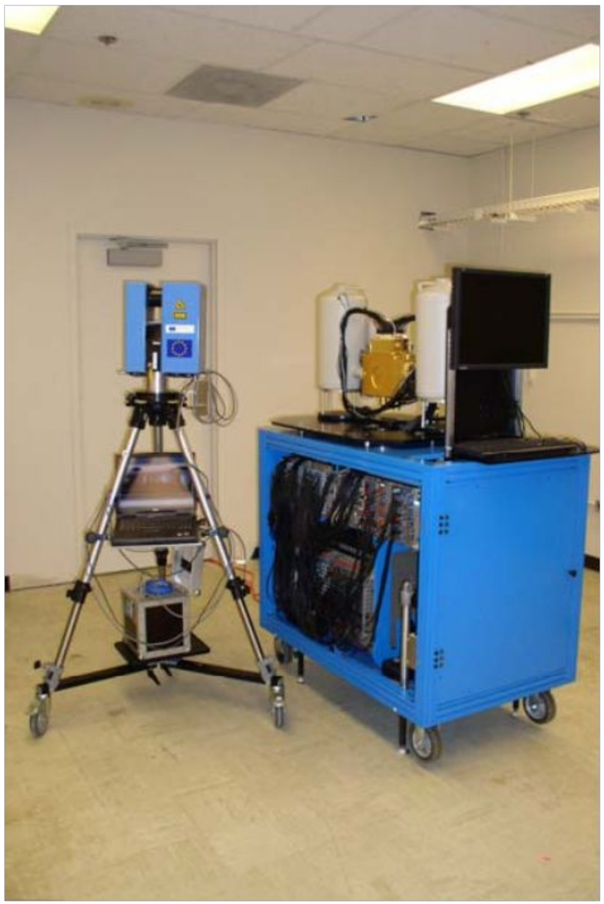
- Volumetric = reconstruction of source distribution in 3 (spatial) dimensions
  - In biomedical imaging, achieved via **tomography**
    - Multiple 2D projections from perspectives around a central image space
    - Having small, well-characterized image space is key!
- Volumetric imaging would be nice for many nuclear security, safeguards, and environmental applications as well
  - Measurement environments that vary hugely in scale (mm → km), complexity, and accessibility

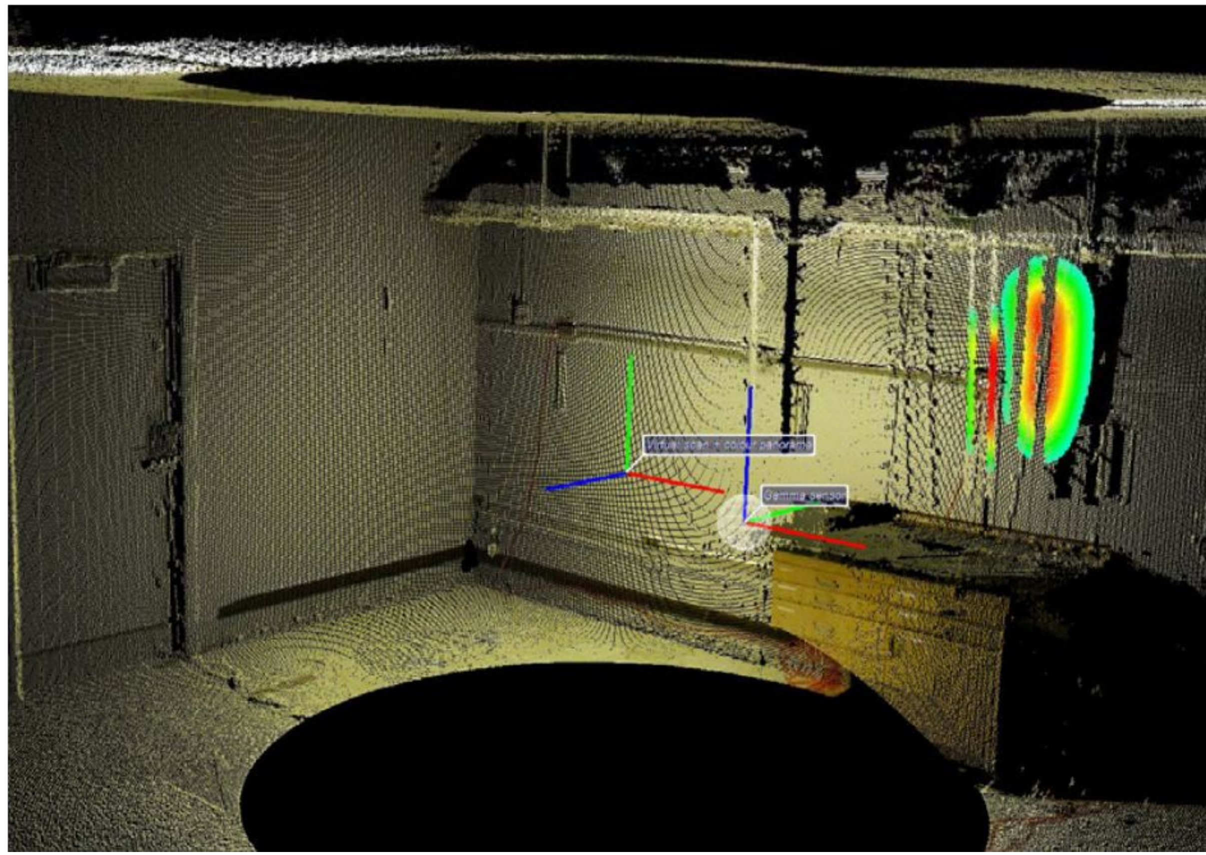


Description of 3D imaging problem as is relevant for many nuclear security applications. From K. Vetter et al 2018

Volumetric Imaging

- Value of 3D reconstruction recognized for many applications
    - Combine conventional gamma-ray imaging with 3D models of environment

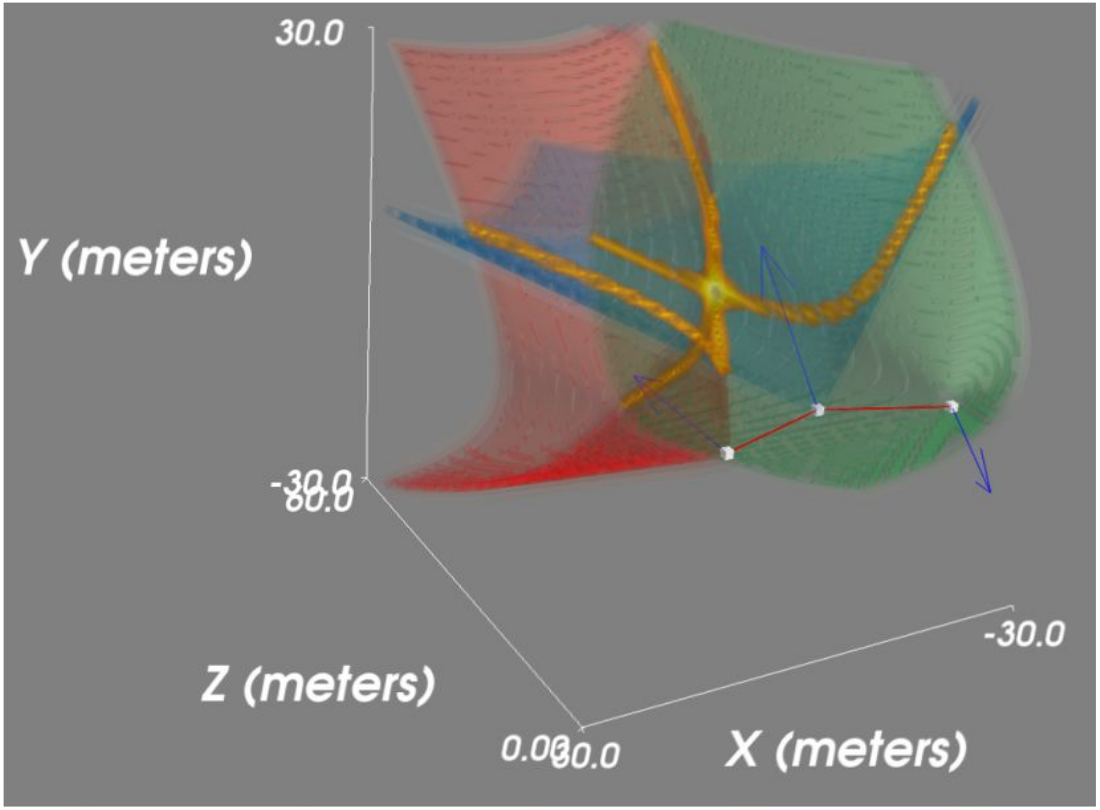




Compton imaging with CCI of Eu-152 line source in pipe mockup of a nuclear facility. From Mihalescu et al 2006

Depth Sensitivity - Triangulation

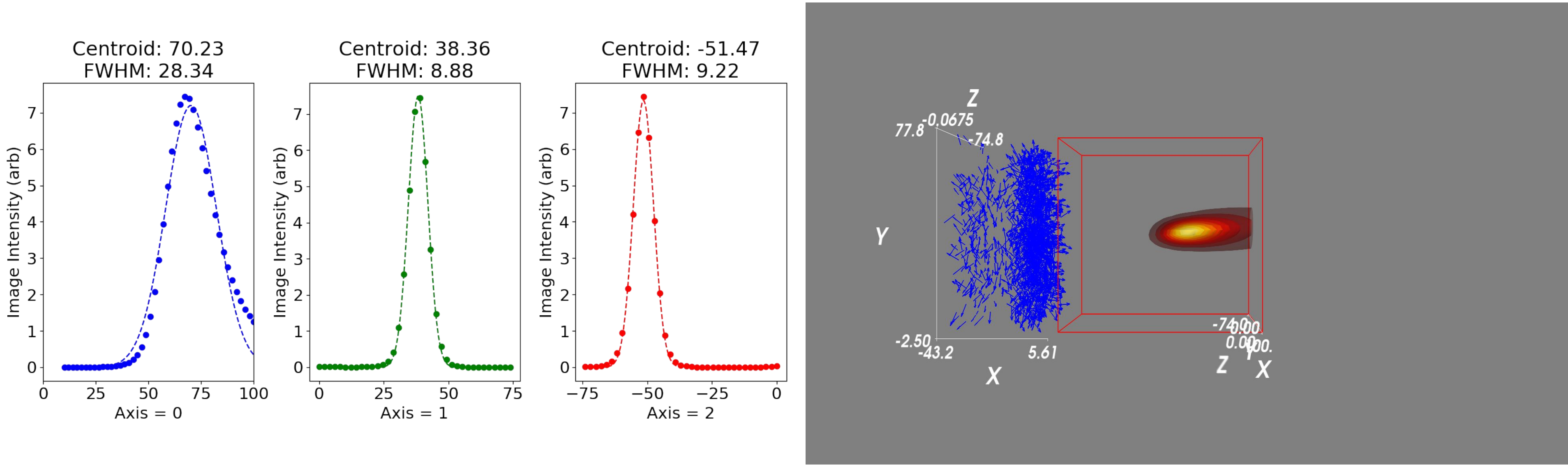
- Can’t rely on traditional tomography for general 3D imaging applications
  - Ability to acquire necessary projections may be limited
- Depth sensitivity from the principle of triangulation



Visualization of principle of triangulation for far-field Compton imaging. The white squares represent different detector locations in space from which Compton cones were reconstructed. From thesis

Triangulation

- Depth resolution limited by triangulation baseline
  - $\Delta Z \sim Z^2/b$; Z = distance to object, b = triangulation baseline

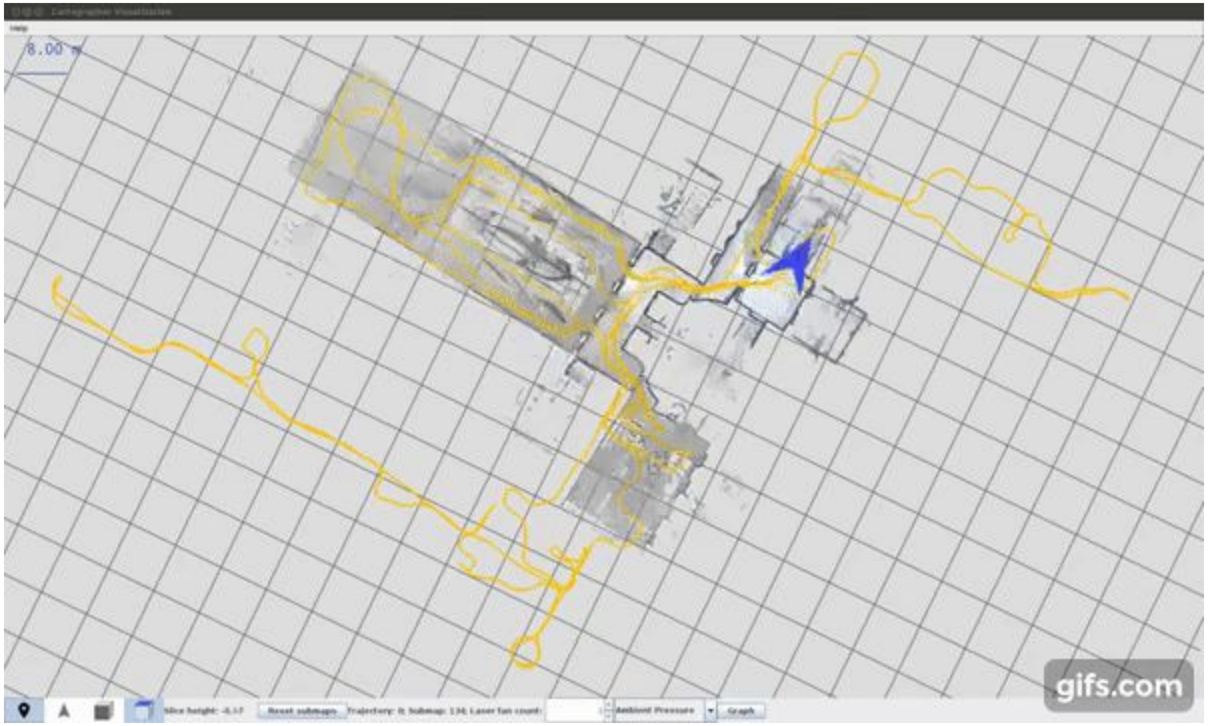


Example: Limited depth resolution in near-field Volumetric reconstruction. In nearfield, triangulation baseline set by dimensions of detector (74 cm for CCI-2)

- Triangulation-based volumetric reconstruction for arbitrarily large environments requires **mobile detector operation** to provide sufficient **triangulation baseline**

Mobile Imagers for Volumetric Imaging

- Need to determine the **location and orientation** (i.e. 6D pose) of gamma-ray imager as it is moved throughout the measurement environment
  - Solution: **Simultaneous Localization and Mapping (SLAM)**



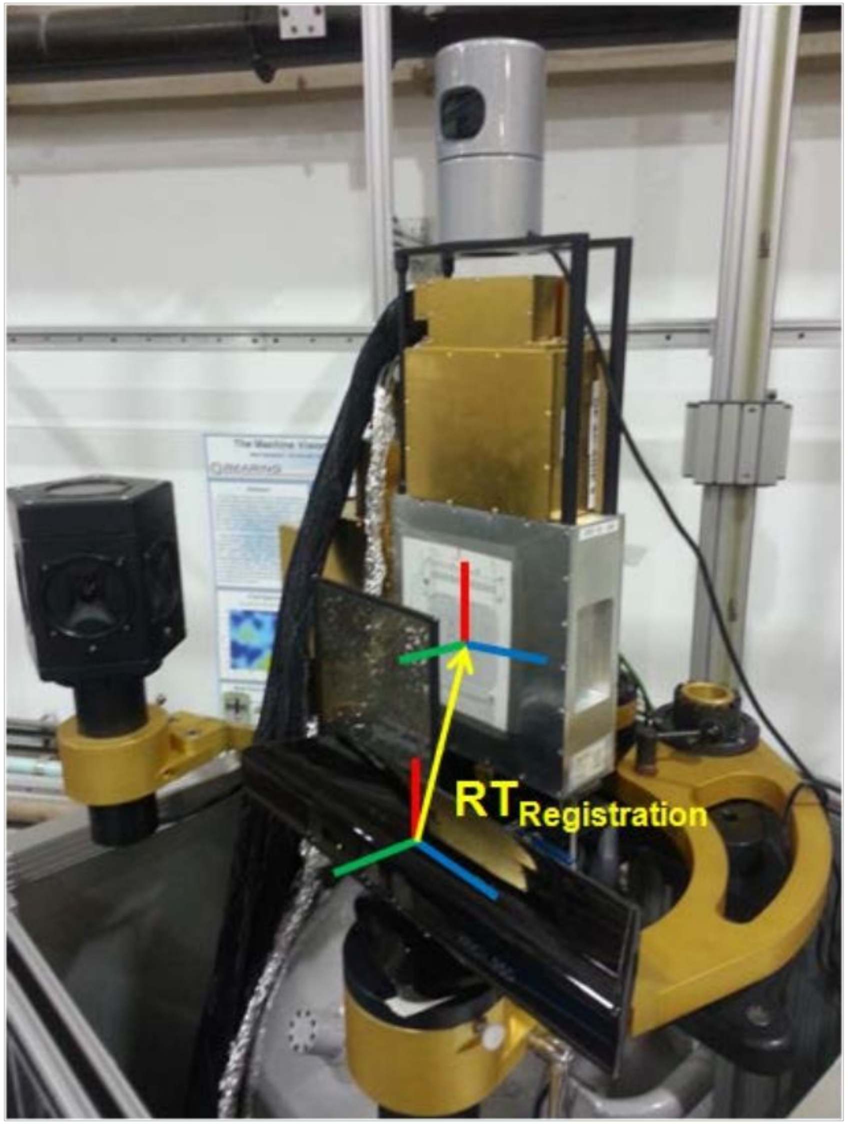
https://www.youtube.com/watch?v=DM0dpHLhtX0&feature=youtu.be

SLAM

- SLAM algorithms provide simultaneous estimates of **sensor pose** and a **model of the measurement environment**
  - Pose estimate necessary for mobile detector operation
  - “Scene” model can be directly incorporated into the gamma-ray image reconstruction
- Many, many different SLAM approaches based on different sensors and algorithms
  - For our purposes, tend to focus on **real-time** SLAM approaches that provide a **3D model** of the environment
  - Some (open source) examples:
    - RGBDSlam: Real-time 3D SLAM based on RGBD camera (e.g. Microsoft Kinect)
    - LSD-SLAM: Real-time monocular SLAM
    - Google Cartographer: Real-time SLAM framework; can provide robust 3D SLAM with LiDAR & IMU

Integration: SLAM + Gamma-ray Imagers

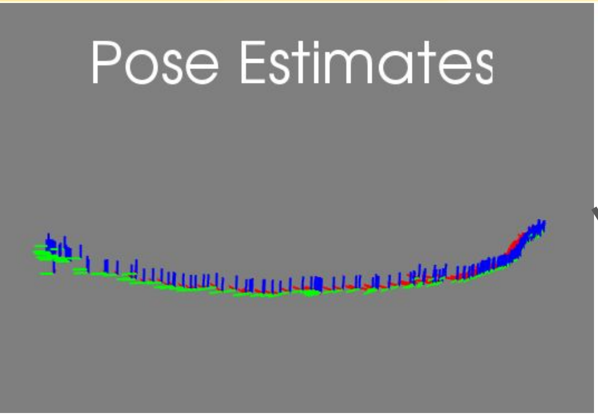
- SLAM provides RT pose estimate of **SLAM sensor**
    - Need estimate of 6D pose of gamma-ray imager!
- Time synchronization
    - Correlate gamma-ray interaction data to most recent pose estimate
- Sensor registration
    - Determine relative transformation between SLAM and Gamma-ray imager coordinate frames



Example: CCI-2 + RGBDSlam

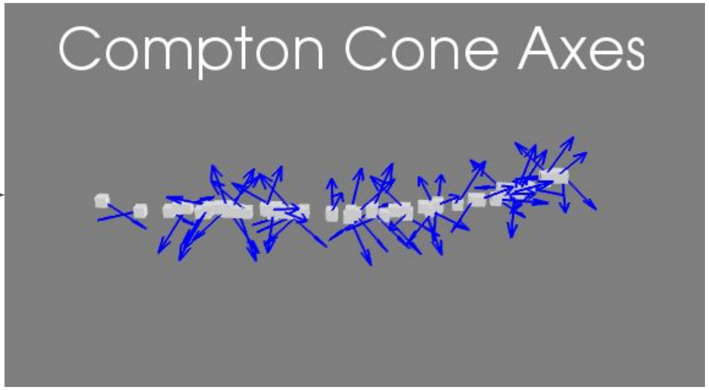
Approx trajectory of CCI-2 cart

Example: CCI-2 + RGBDSlam



Pose of Kinect, from RGBDSlam

Synchronization & Registration



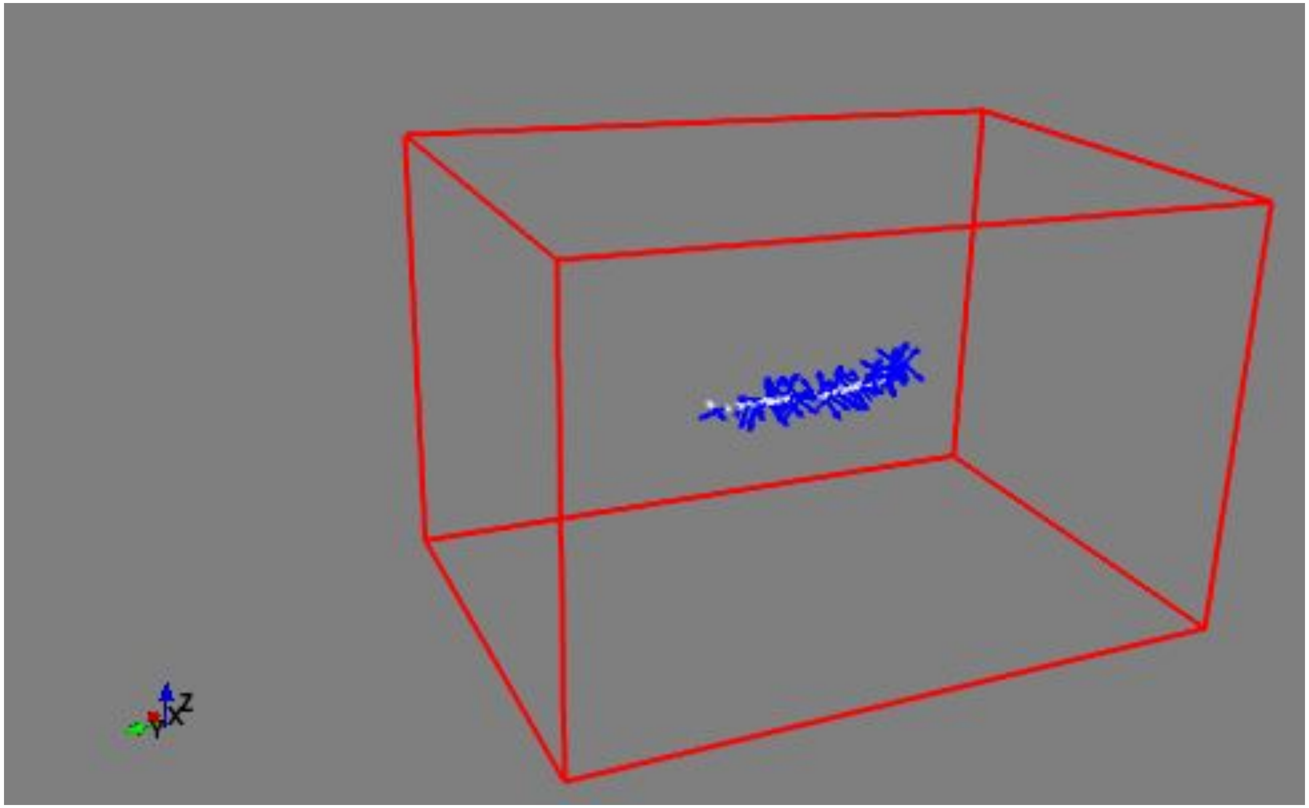
Gamma-ray Interaction Data (Compton cones)

From CCI-2

Example: CCI2 + RGBDSlam

- Construct volumetric, discretized image space
  - Use model, pose estimates, or some other means of defining the bounds of the image space
  - Discretize the space

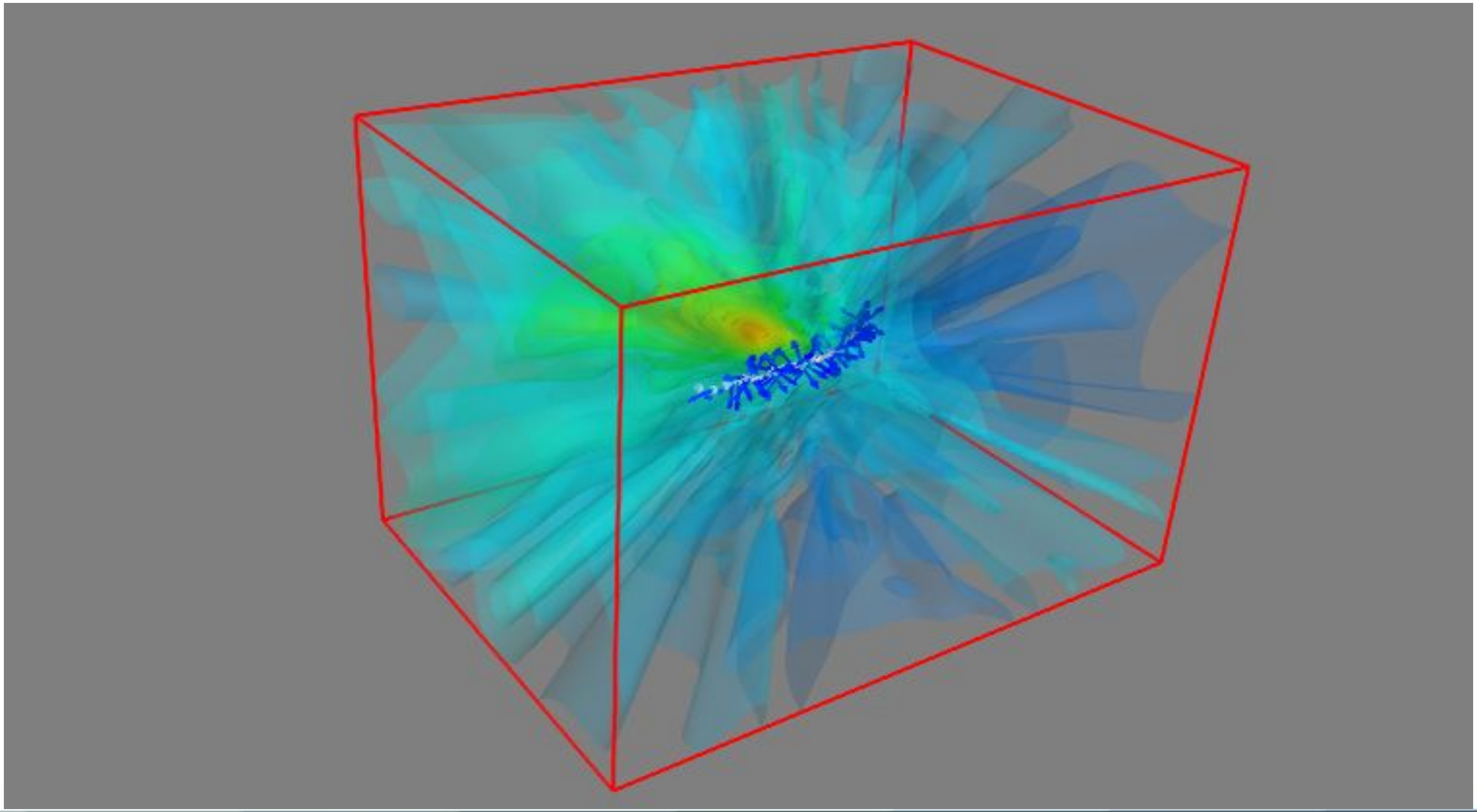
Image bounding box determined by maximum distances @ a 4m radius from each point along track. BBX subsequently discretized into 10 cm voxels



Example: CCI-2 + RGBDSlam

- Backproject Compton cones into image space
  - This measurement: only 72 cones

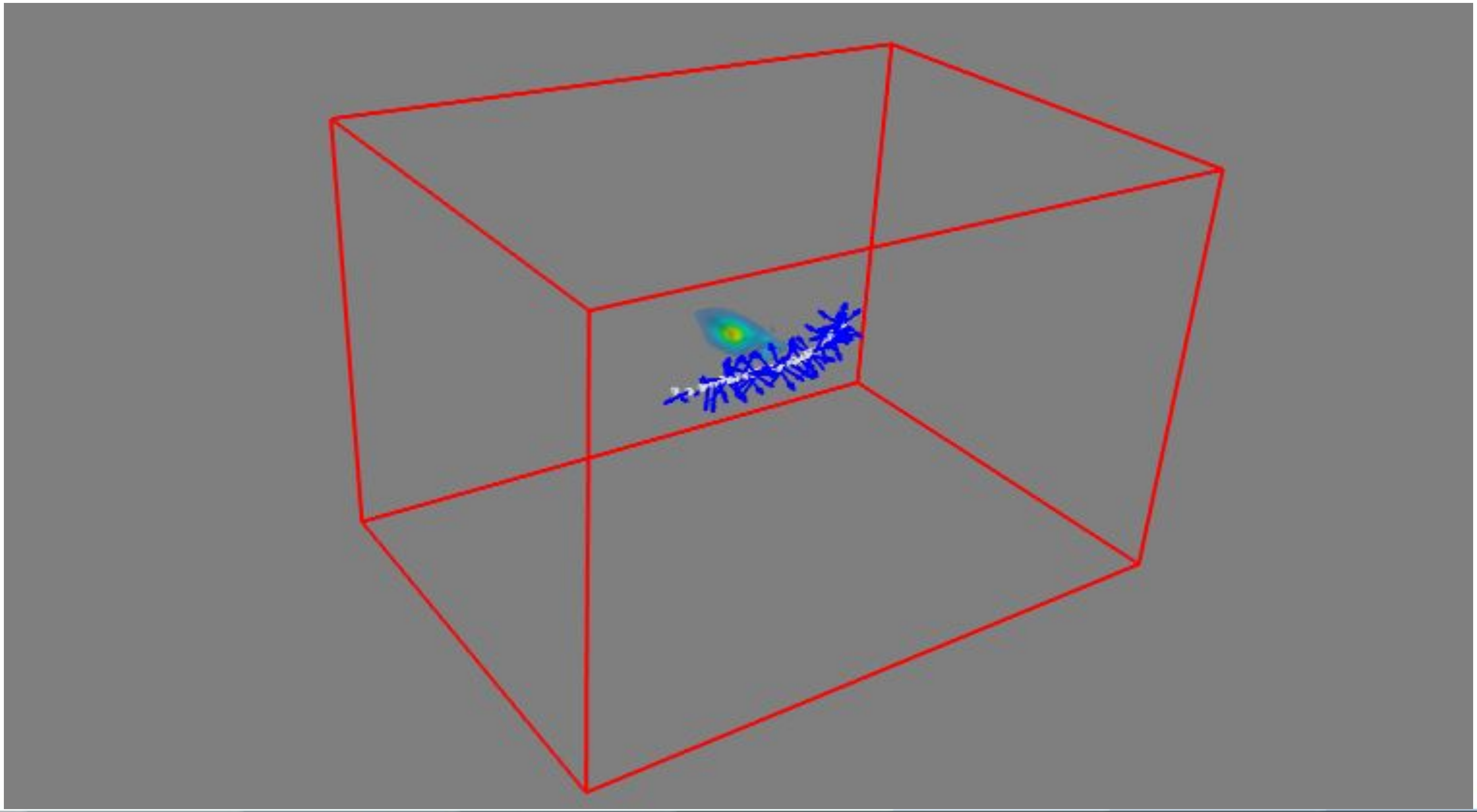
Result of backprojection of 72 Compton cones into voxellized image space. Intensity of backprojection visualized with contour surfaces



Example: CCI-2 + RGBDSlam

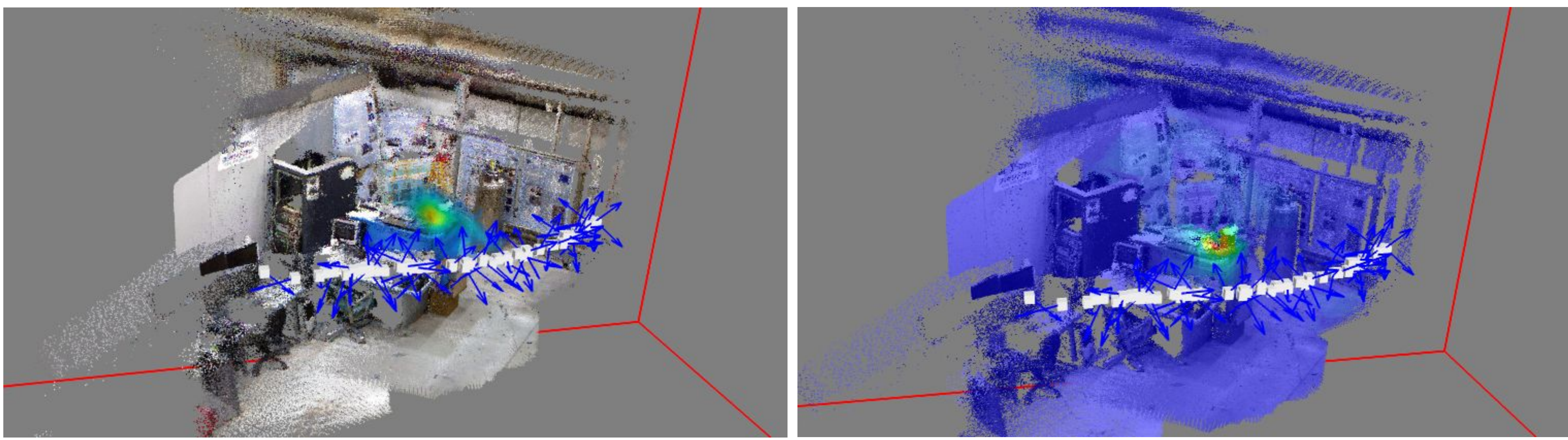
- Backproject Compton cones into image space
  - This measurement: only 72 cones
  - Can subsequently perform EM to improve image

Same result, using the backprojection as the 0th iteration of the Poisson ML-EM algorithm (see reading assignment 8) and running for 10 iterations



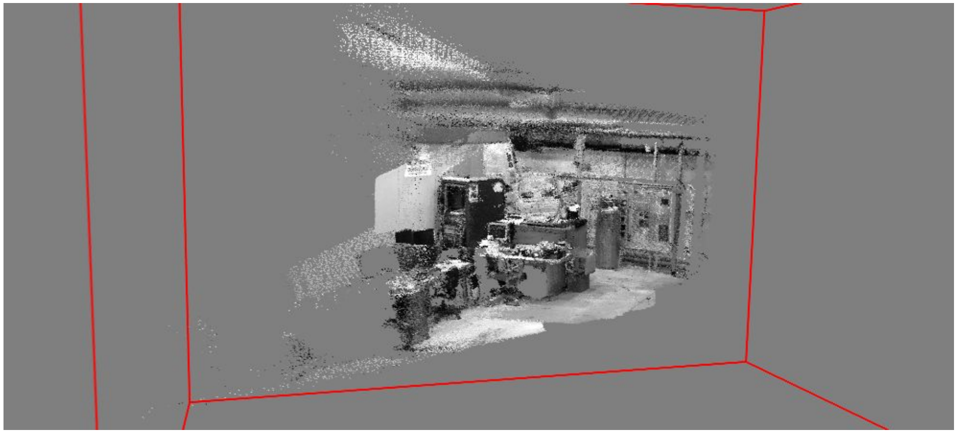
Example: CCI2 + RGBDSlam

- Visualization: Plot the 3D model to provide context for the gamma-ray image
    - Plot 3D contour surfaces in the same pane as 3D model (left)
    - Colorize 3D model with intensity values from 3D gamma-ray image (right)



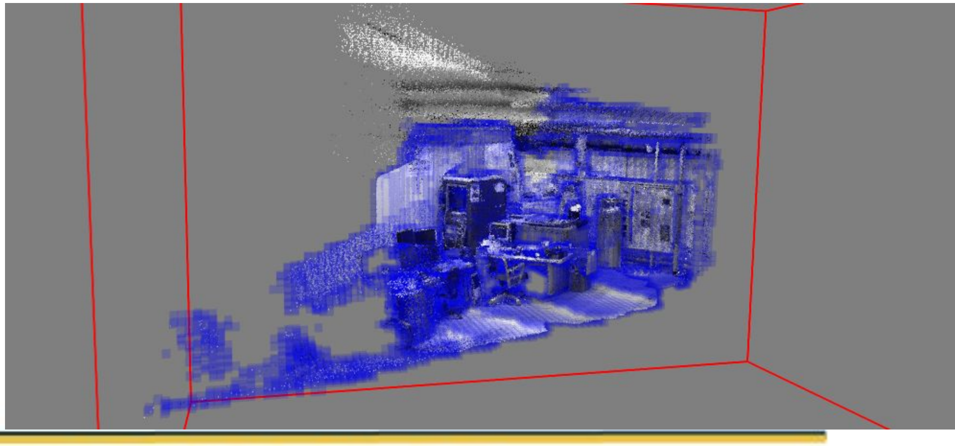
Scene Data Fusion: Incorporate Contextual Information into the Gamma-Ray Image Reconstruction

- E.g. - Assume gamma-ray sources are on the surfaces of objects in the scene
  - Occupancy constraint: can use 3D model to constrain image space!
    - Reduces computational burden (memory, compute time)
    - Aids in real-time gamma-ray image reconstruction



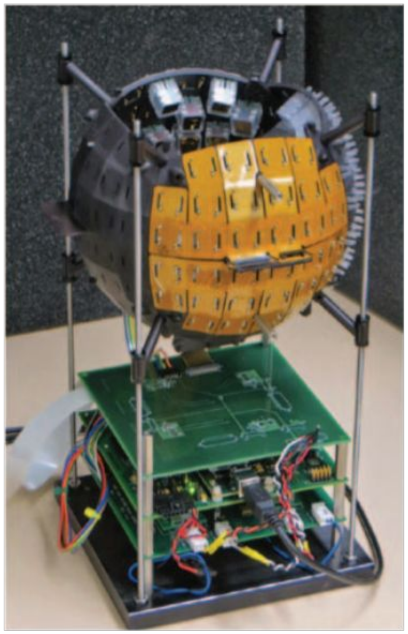
Original image space = 90 x 125 x 82 → 922,500 voxels (point cloud model included for context)

Image space w/ occupancy constraint: 16,460 voxels - greater than 50x reduction in image space!

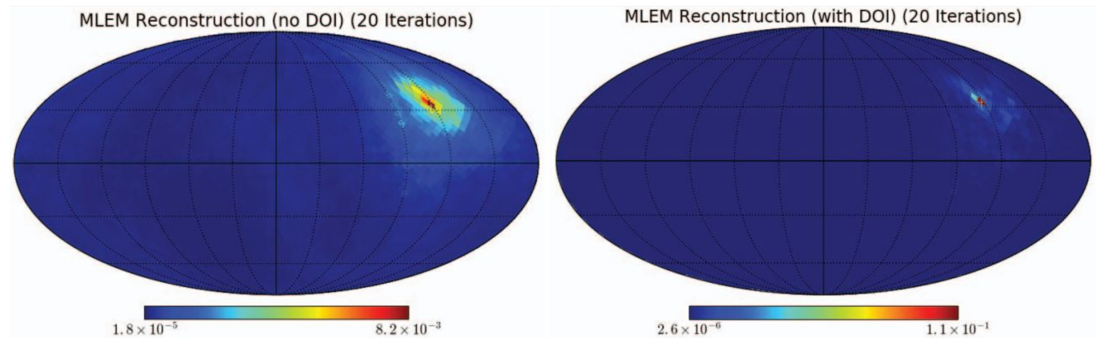


Gamma-Ray Imaging & Mapping with Mobile Detection Systems

- Previous example was Compton imaging, but this approach works for other imaging modalities as well
    - E.g. spherical (4π) coded aperture



(b)



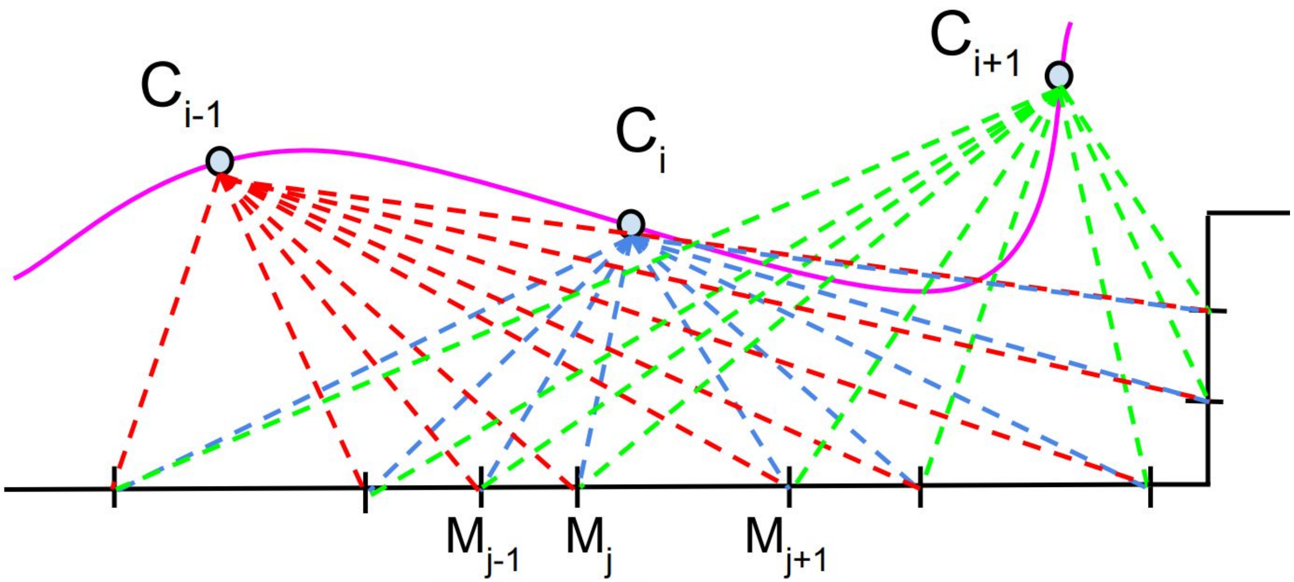
(e) (f)

From Dan Hellfeld's presentation @ IEEE 2017

- Proximity imaging
    - Mobile detector operations → radiation mapping with non-imaging detectors

Proximity Imaging

- Limitations of static imaging: distance to the source
  - Photons incident on imager ~ $1/R^2$
- Mobile operation → use distance modulation to your advantage
  - Resolution scales with distance-of-closest-approach to source
    - Strong case for unmanned platforms (e.g. ground/air) which may be able to get nearer to sources in more complex environments



Proximity imaging formulation; M represents the image space (map) while C represents the detector response (CR, ROI CR, etc.). The dotted lines illustrate the distance connecting the various image space points to the sampling points along the detector track (magenta). **From R.T. Pavlovski (publication under review)**