Signal Formation in Semiconductor Detectors

- Integrate **induced current** from **charge motion** to recover **induced charge** ∝ initial # charge carriers (energy dep.)
  - Shockley-Ramo theorem greatly simplifies analysis of induced current/charge
    - Calculate signal shapes for various detector/electrode geometries
    - Position-sensitive gamma-ray detectors, pulse-shape analysis
- Applications in semiconductor detectors
  - Compound Semiconductors with poor hole mobility
    - Pixelated anode, coplanar grid electrodes, virtual Frisch grid
  - Small-pixel effect
    - Lateral position sensitivity
    - Sub-pixel position resolution: “image charges”
    - Depth determination (ΔT_{50}, Cathode/Anode ratio)
  - Event selection based on signal shape
    - E.g. PPC for rare-event searches
  - Exotic electrode segmentation schemes

Induced Signal

- For radiation detectors based on detection of direct ionization (gas det., semiconductors): signal is due to the **motion of charge carriers**
  - Thus, the signal ultimately depends on:
    i. **Position of charge carriers as a function of time**
    ii. **Coupling of charge carriers to sensing electrodes**

I. Depends on electric field and mobility
  - $p(t) = p_0 + v_d * t$
    - p = position as function of time
    - v_d = *carrier drift velocity* = $\mu * E$
      - Note that electric field (*E*) is likely also position dependent!
  - *E* →Poisson equation: $\nabla^2\varphi = \varrho/\varepsilon$

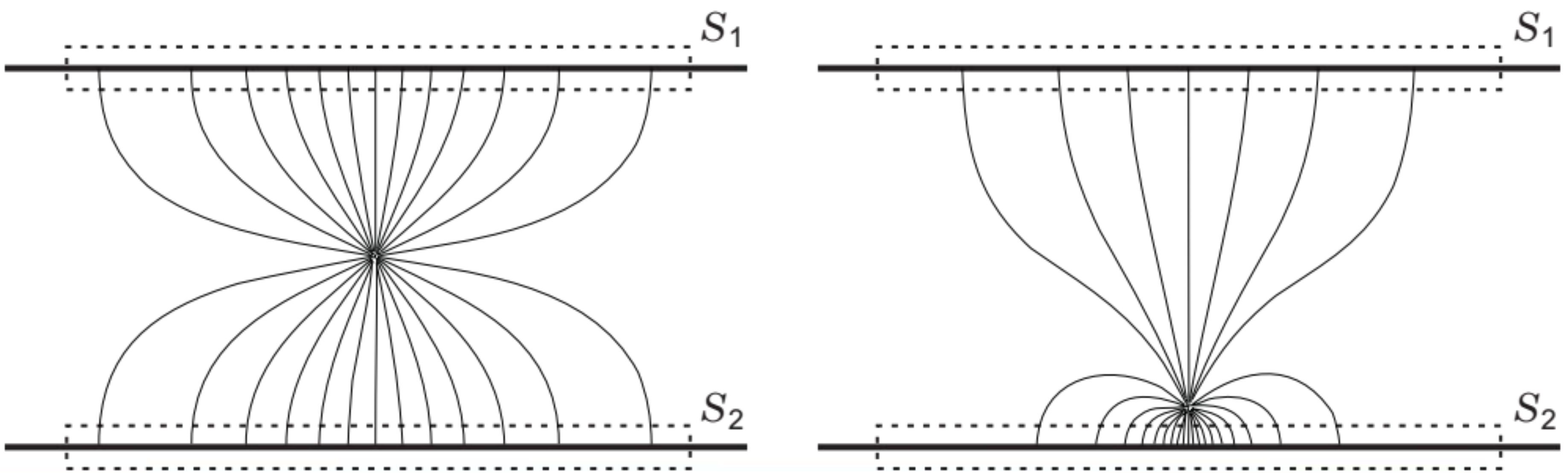
II. Weighting potential/field →Laplace equation: $\nabla^2\varphi_w = 0$
  - Shockley-Ramo theorem

Shockley-Ramo Theorem

- Why do we make such a big fuss over it?
  - Greatly simplifies calculation of induced charge:
  - Without SR: calculate instantaneous *E* from *q* at every point along trajectory & integrate E over electrode surface:

$$Q = \oint_S \varepsilon \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S}$$

Spieler Fig. 2.28



Shockley-Ramo Theorem

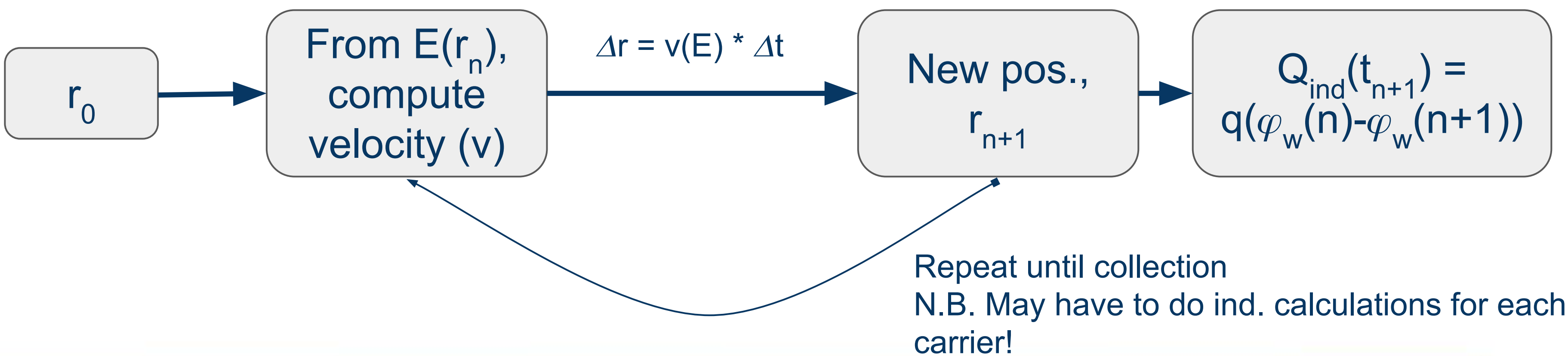
- Why do we make such a big fuss over it?
  - With SR theorem, can describe coupling of charge to any electrode much more simply:
    - $i_{induced} = q\ \mathbf{v} \cdot \mathbf{E}_{\mathbf{weighting}}$ ← Weighting **field**
    - $Q_{induced} = q\ \Delta\varphi_{\mathbf{weighting}}$ ← Weighting **potential**
  - See **Spieler sec. 2.5** for derivation (or RA1, RA6)

- Solve for weighting field ($E_{weighting}$) and weighting potential ($\varphi_{weighting}$) via Laplace equation (ignoring static space charge)
  - $\nabla^2\varphi_{weighting} = 0$ $\qquad E_{weighting} = -\nabla\varphi_{weighting}$
  - Boundary conditions
    - Potential at electrode of interest = 1
    - Potential at all other electrodes = 0

Applying Shockley Ramo

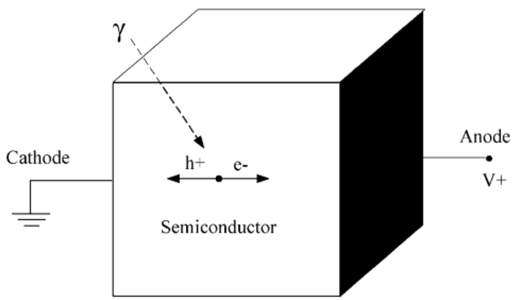
- How do we get from Electric & Weighting fields/potentials to signals?

1. State your assumptions! E.g:
   a. Point-charges (ignore electron cloud)
   b. Carrier velocity (see Knoll) $v = \frac{\mu_0 E}{(1 + (E/E_0)^{\beta})^{1/\beta}}$
   c. Many others…
2. Solve for Weighting and Electric potentials (and fields) for given geometry & electrode configuration
3. Select an initial position, r_0

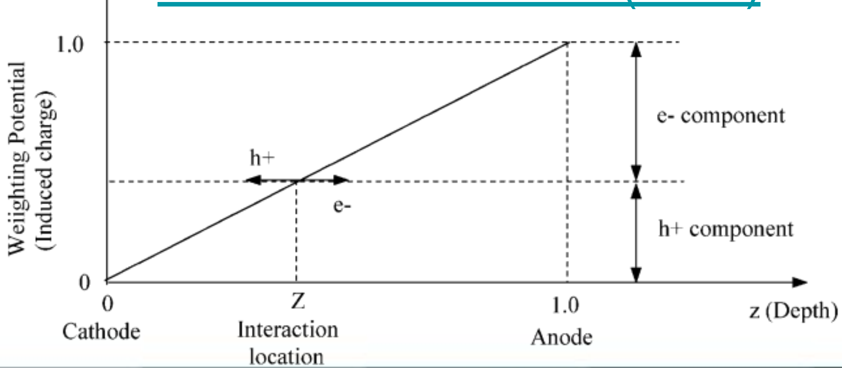


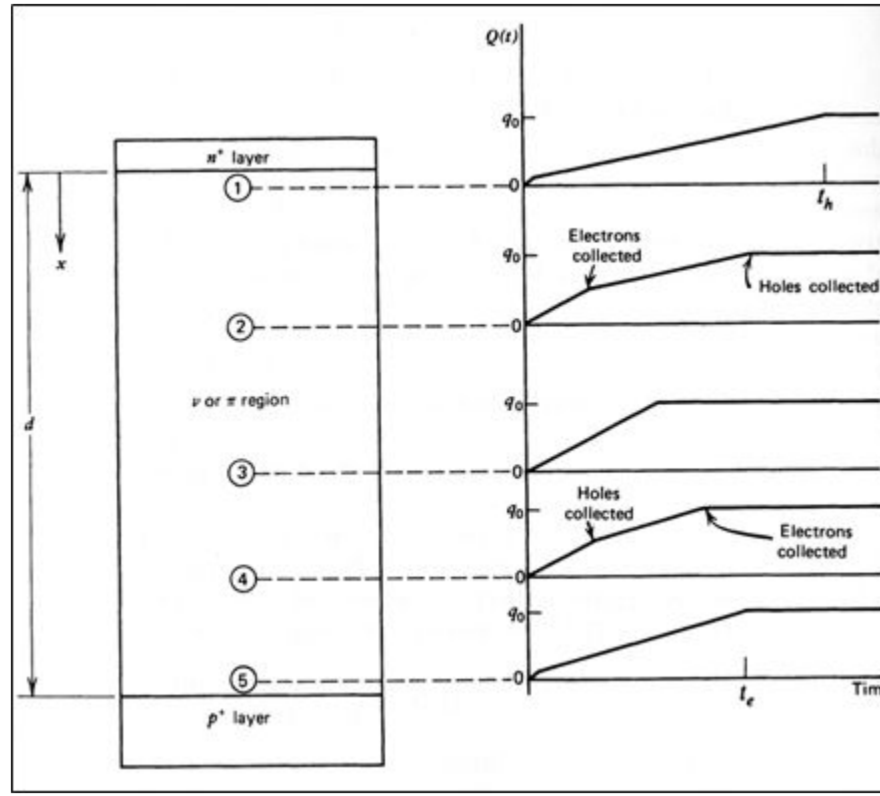
Applying Shockley Ramo

- E_w, φ_w depend only on geometry
- Simple geometries: analytic solutions
  - Planar, 2-electrode geom



[Z. He - Review of SR (RA 6)]





Knoll Fig 12.12

Applying Shockley Ramo

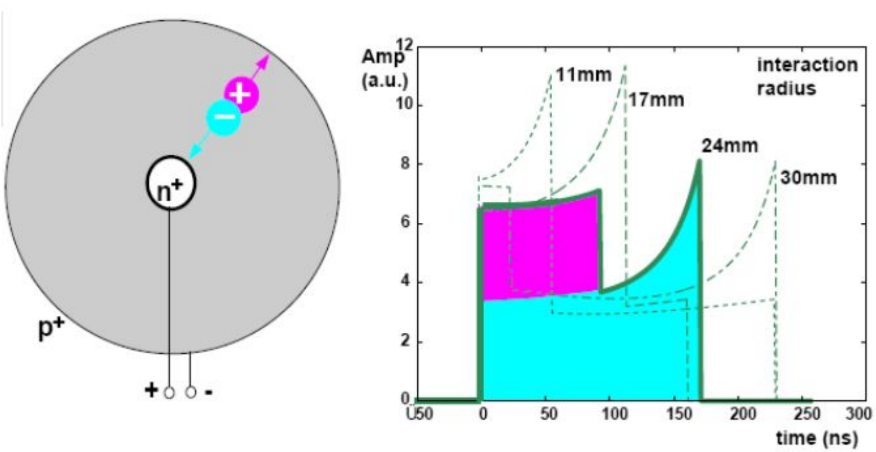
- E_w, φ_w depend only on geometry
- Simple geometries: analytic solutions
  - Through-hole coaxial geometry

$$Q(t) = Q^-(t) + Q^+(t)$$

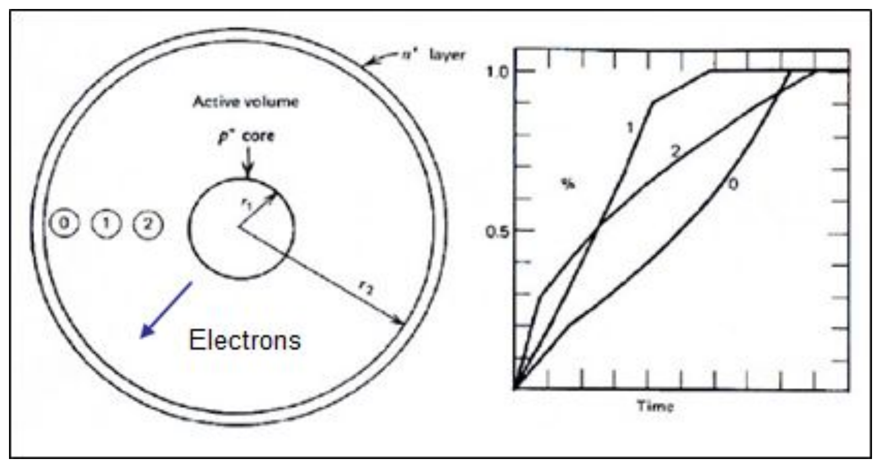
Knoll ch. 12

$$Q^+(t) = \frac{q_0\alpha}{V_0}\left[r_0^2 - r_h^2(t)\right] + \frac{q_0\beta}{V_0}\ln\frac{r_0}{r_h(t)}$$

$$Q^-(t) = \frac{\Delta E^-}{V_0} = \frac{q_0\alpha}{V_0}\left[r_e^2(t) - r_0^2\right] + \frac{q_0\beta}{V_0}\ln\frac{r_e(t)}{r_0}$$



J. Gerl et al AGATA Technical Proposal Sept. 2001

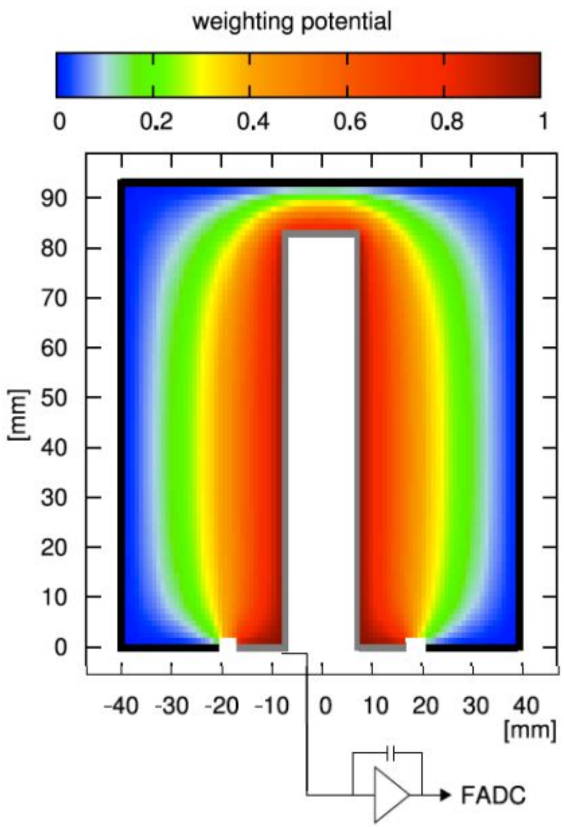


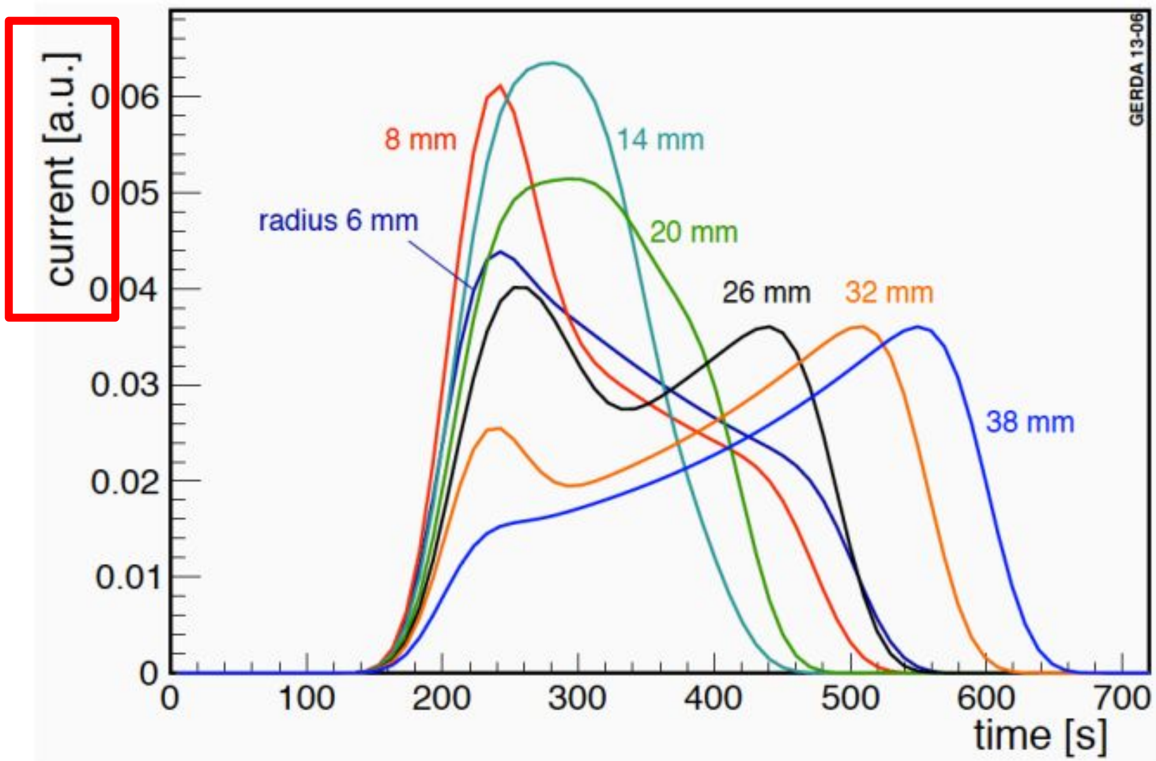
Knoll fig. 12.13

Fields and Potentials

- For more complex geometries / electrode structures → numerical solutions for electric & weighting potentials/fields

E.g. Closed-Ended coaxial HPGe detector



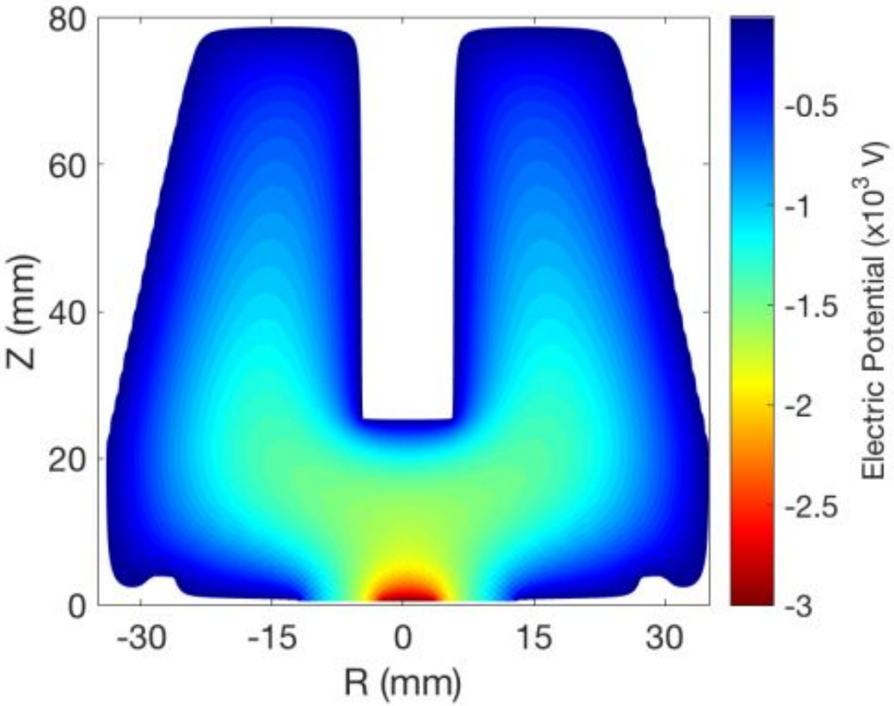


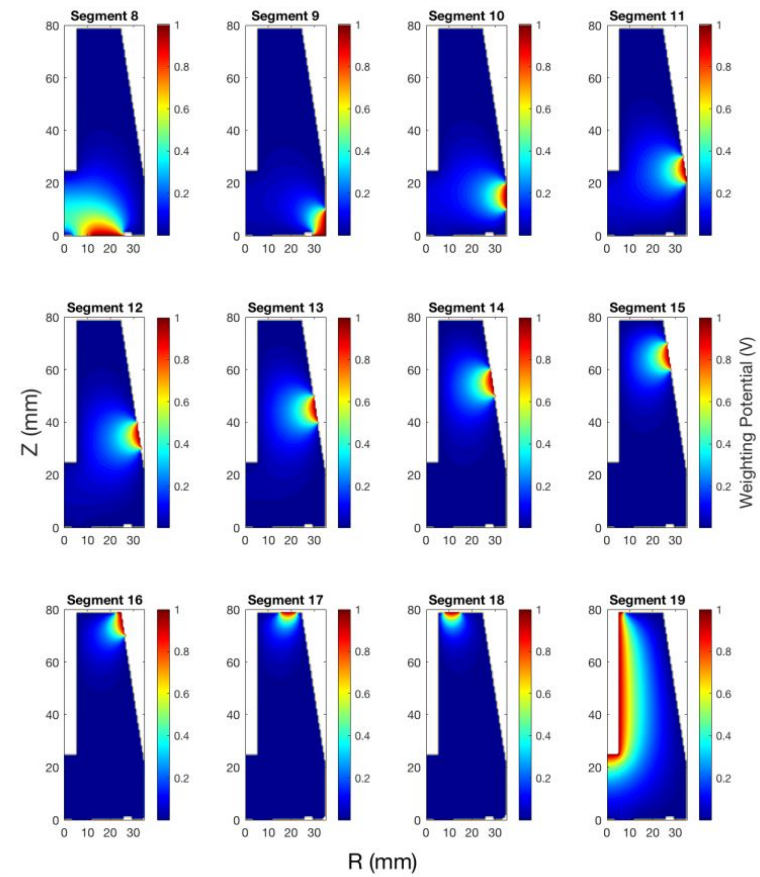
Images from M. Agostini - Pulse Shape Discrimination for GERDA Phase I Data

Fields and Potentials

- For more complex geometries / electrode structures → numerical solutions for electric & weighting potentials/fields

  E.g. Segmented interted coax design (cf. guest lecture from Marco Salathe)





Images from [J. Wright: Status and Expected Performance of the SIGMA detector]

Case Study: Detector Designs

- Use knowledge of signal induction process in semiconductors to design detectors with “special” capabilities/characteristics
  1. Single-polarity charge sensing
  2. Position-sensitive gamma-ray detectors
  3. Event selection/rejection based on pulse-shape analysis

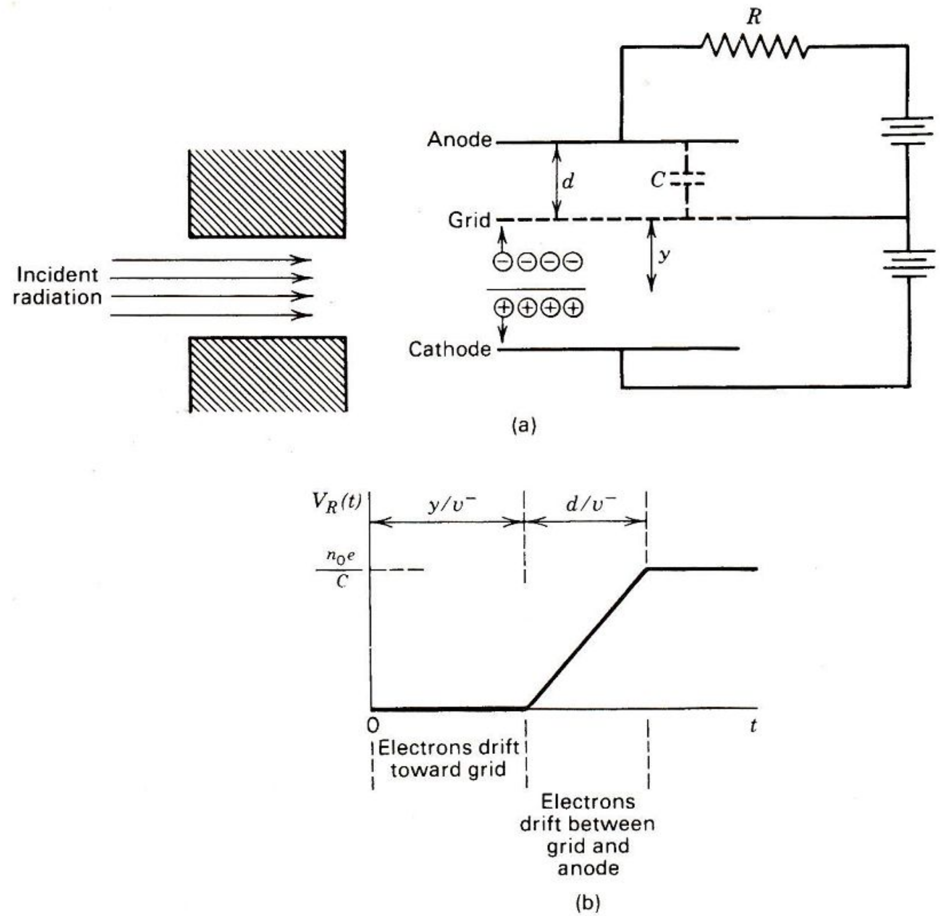
Single-Polarity Charge Sensing (SPS)

- In what scenario is single-polarity charge sensitivity useful?
  - When carriers have vastly different mobilities or lifetimes
    - Gasses
    - Compound Semiconductor Materials

Material	Z	Density [g/cm^3]	Bandgap [eV]	W [eV]	ρ at 25°C [Ωcm]	μ_e [cm^2/V s]	μ_h [cm^2/V s]	τ_e [s]	τ_h [s]	$\mu\tau_e$ [cm^2/V]	$\mu\tau_h$ [cm^2/V]
Ge	32	5.32	0.7	2.96	50	3900	1900	$>10^{-3}$	10^{-3}	>1	>1
Si	14	2.33	1.1	3.62	$<5\times10^4$	1400	480	$>10^{-3}$	2×10^{-3}	>1	~1
Diamond	6	6.0	5.4	13.25		2000	1600	10^{-8}	$<10^{-8}$	2×10^{-5}	$<2\times10^{-5}$
CdTe	48,52	6.2	1.44	4.43	10^9	1100	100	3×10^{-6}	2×10^{-6}	3.3×10^{-3}	2×10^{-4}
CdZnTe	48,30,52	~6.0	~1.8	~5.0	10^{11}	1350	120	10^{-6}	5×10^{-8}	1×10^{-3}	6×10^{-6}
HgI_2	80,53	6.4	2.1	4.2	10^{13}	100	4	10^{-6}	10^{-5}	10^{-4}	4×10^{-5}
GaAs	31,33	5.32	1.4	4.2	10^7	8000	400	10^{-8}	10^{-7}	8×10^{-5}	4×10^{-6}

Gas Ionization Detectors: Frisch Grid

- $V_{ion} >> v_{e-}$
  - Signal component from ion drift ~10^3 slower than electrons
- Frisch grid
  - Held at intermediate potential between two electrodes
  - Transparent to electrons
- Signal induction at anode due only to electron drift between grid/anode region!



Knoll Fig 12.12

Gas Ionization Detectors: Frisch Grid

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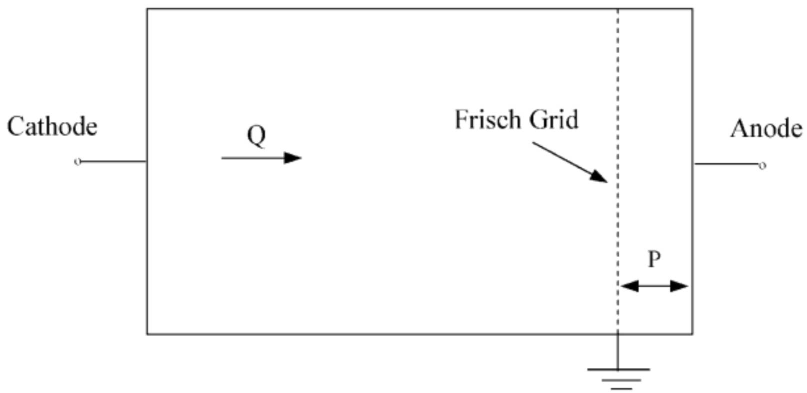
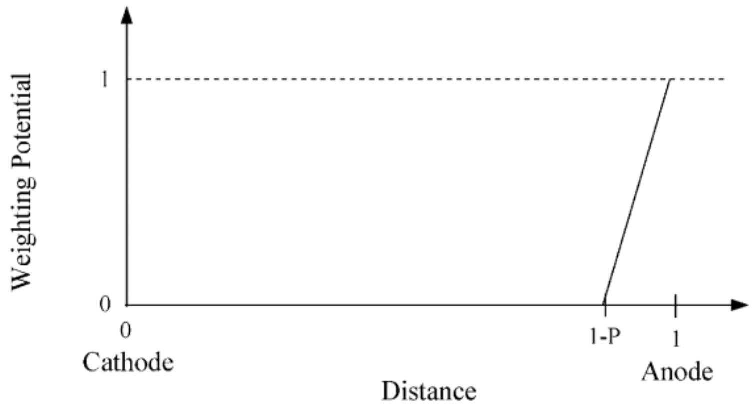
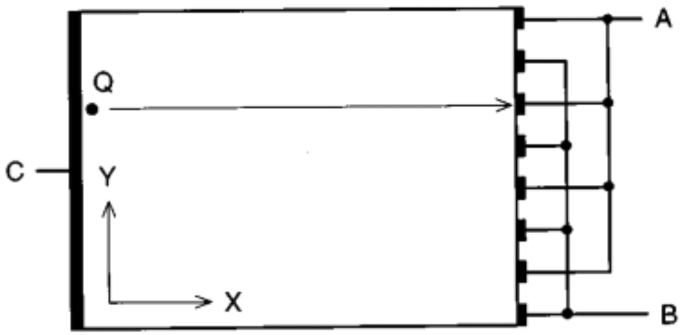


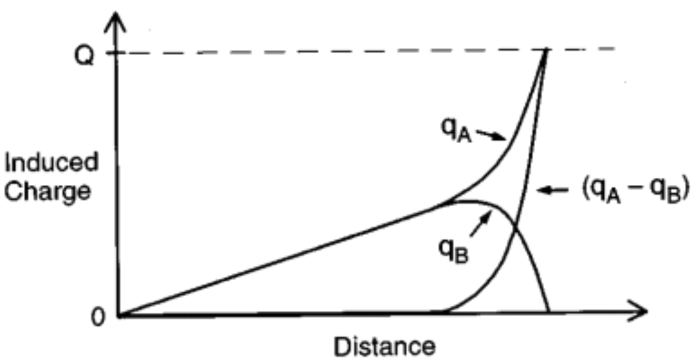


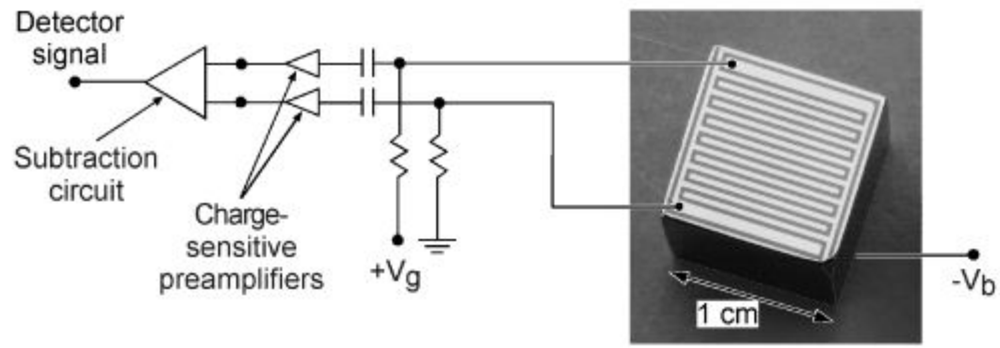
Z. He - Review of SR (RA 6)

SPS in Semiconductor Detectors

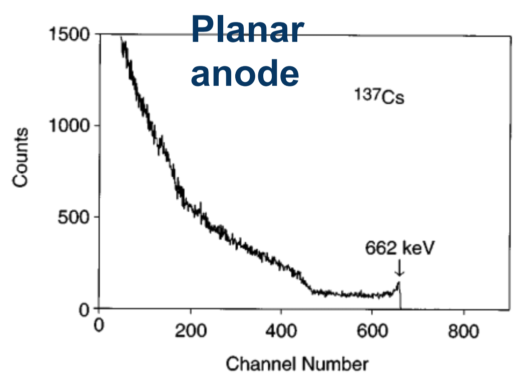
- Example 1: Coplanar grid (CPG) anode
  - “Interdigitated” electrode structure

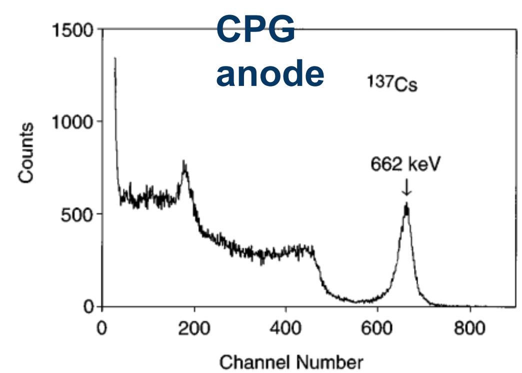






P.N. Luke - LBNL

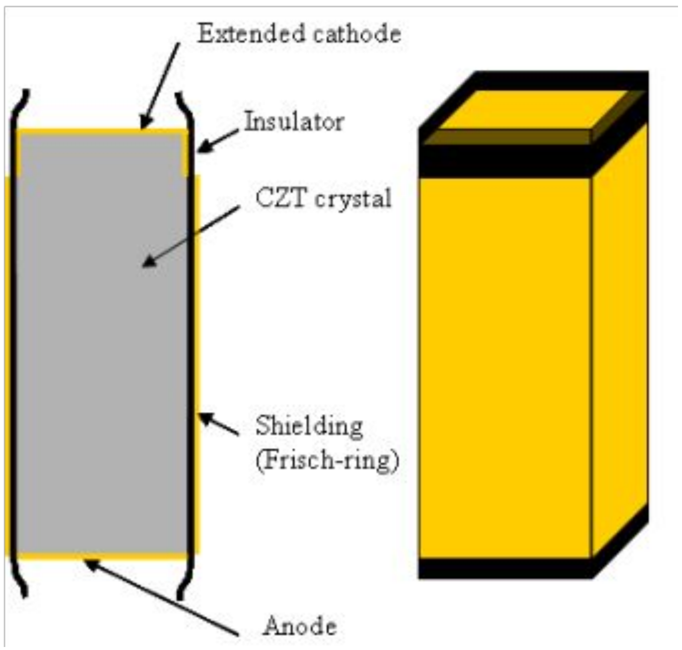


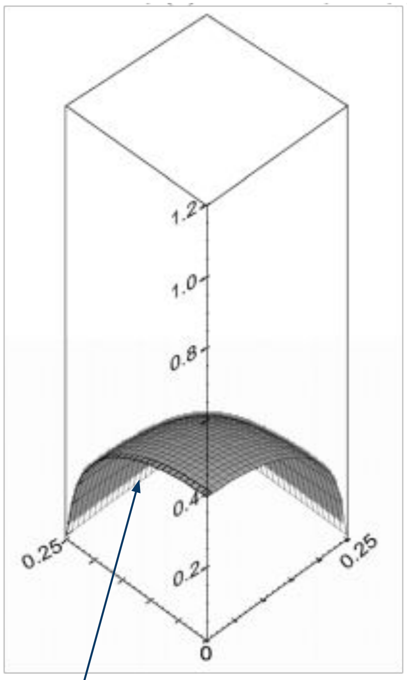


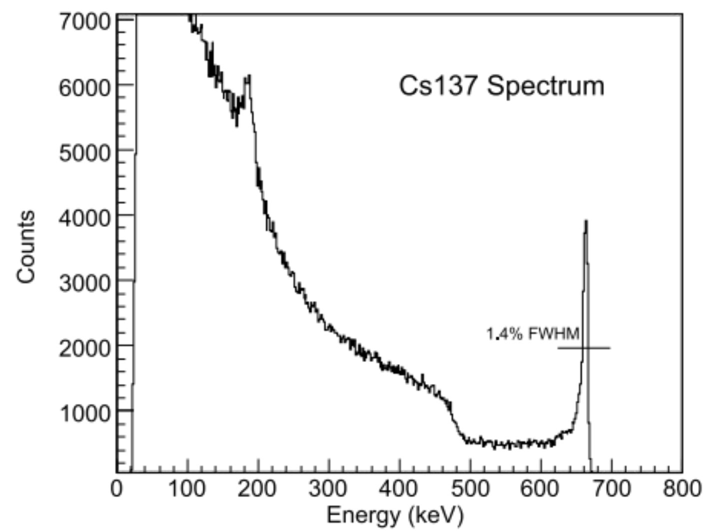
P.N. Luke - Single-Polarity Charge Sensing in Ionization Detectors using Coplanar Electrodes

SPS in Semiconductor Detectors

- Example 2: Virtual Frisch Grid (VFG)
  - Frisch-ring shielding electrode
  - Carrier motion coupled to Frisch ring until very near the anode





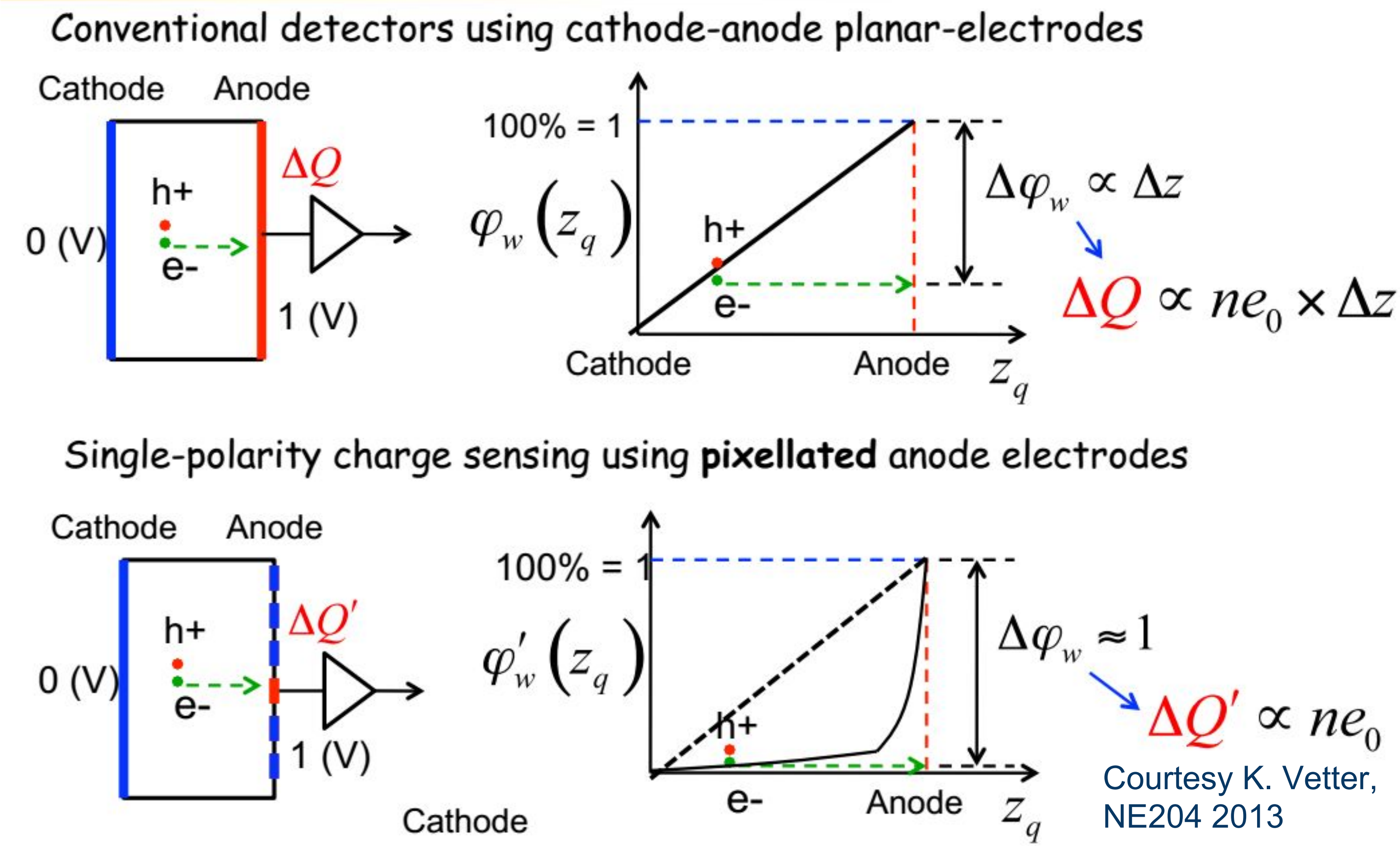


Virtual location of Frisch grid

[Cui, Bolotnikov et al - CZT Virtual Frisch-grid Detector:Principles and Applications]

SPS in Semiconductor Detectors

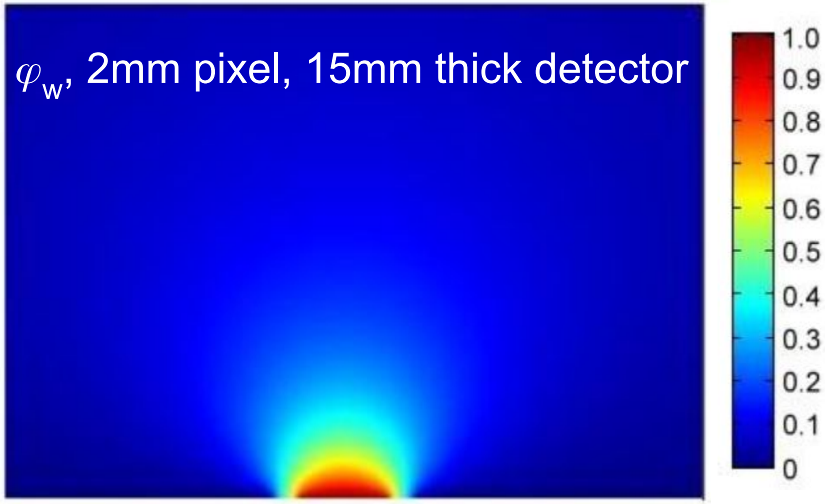
- Example 3: Pixelated Anode
  - Rely on small-pixel effect (more on this in a bit)

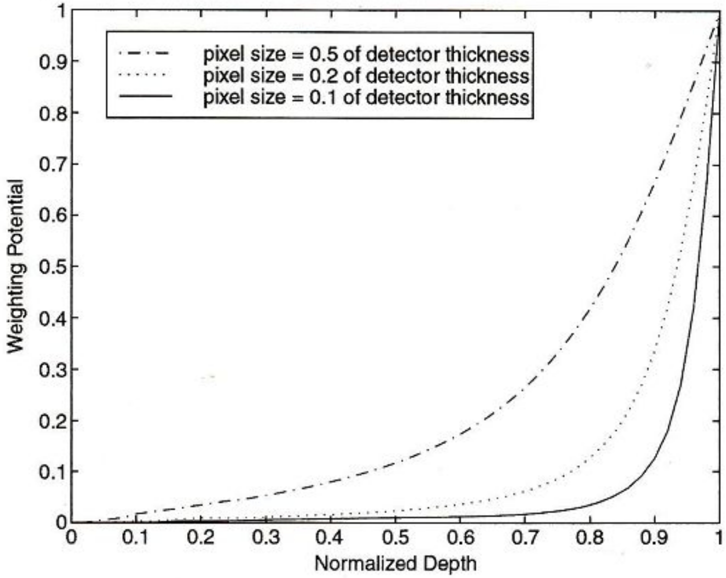


Courtesy K. Vetter, NE204 2013

Small Pixel Effect

- As electrode decreases in size, φ_w extends smaller distance into detector vol.
  - Electrode width / det. Thickness
- Consequences
  - Single-polarity sensitivity (see previous slide)
  - **Position sensitivity**
    - Lateral due to φ_w directly
    - Depth from signal comparison between opposite electrodes

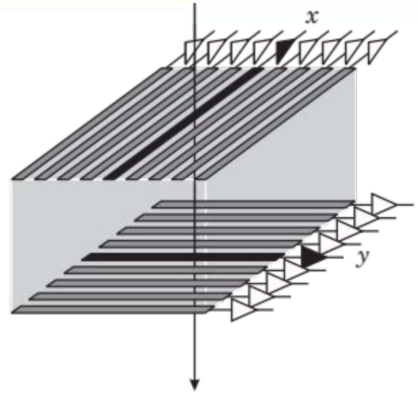




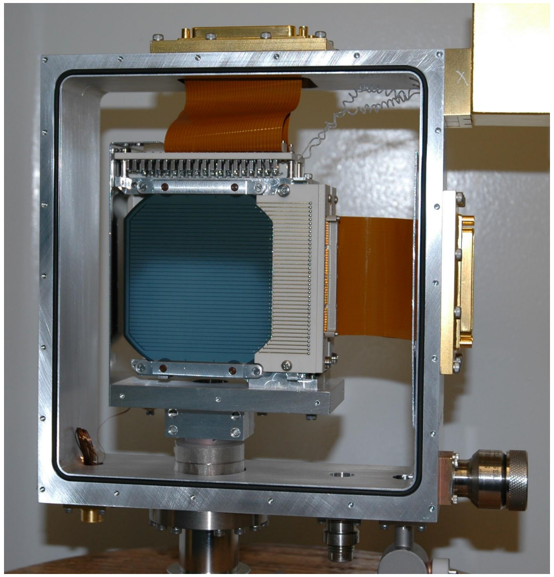
Knoll Fig. D.2

Position-Sensitive Detector Configurations

- Double-sided strip segmented electrodes
  - Each side provides 1D pos. Sensitivity
  - Requires good collection of both carriers (HPGe, thin CdTe)
  - Readout channels for n x n pixels: 2n

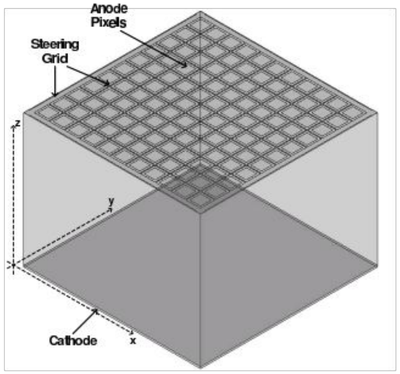


Spieler 1.11

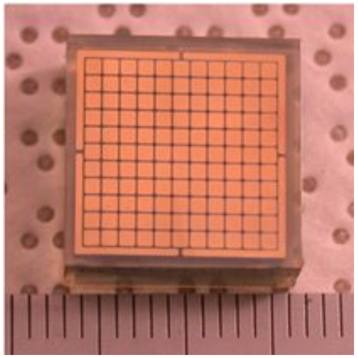


CCI-2 (courtesy M. Amman)

- Pixelated electrode
  - 2D segmentation of one electrode
  - Single-polarity sensitivity (CZT)
  - Readout channels for n x n pixels: n^2



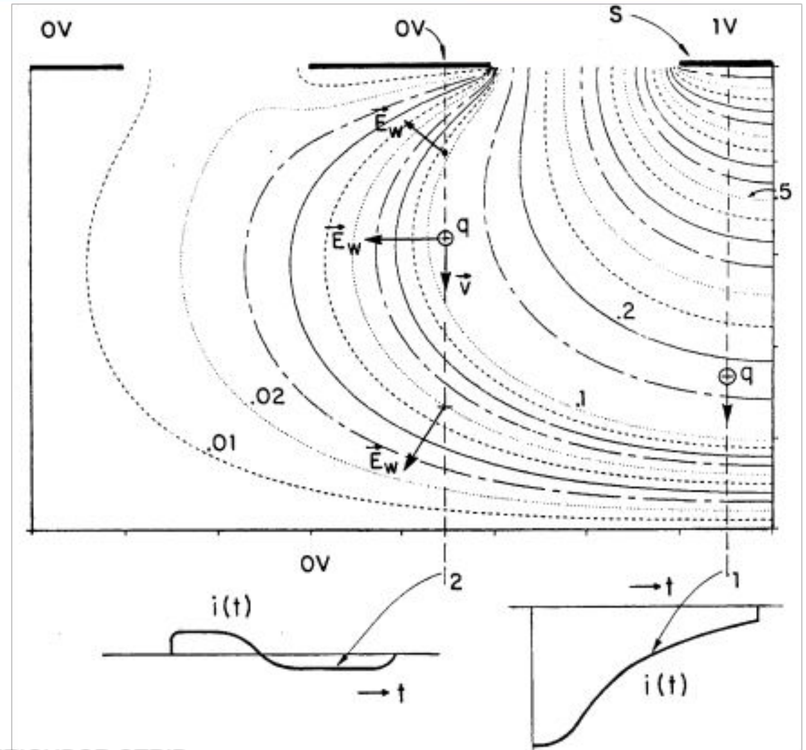
Y. Zhu - DSP Methods for Pixelated 3-D Position Sensitive RTSD



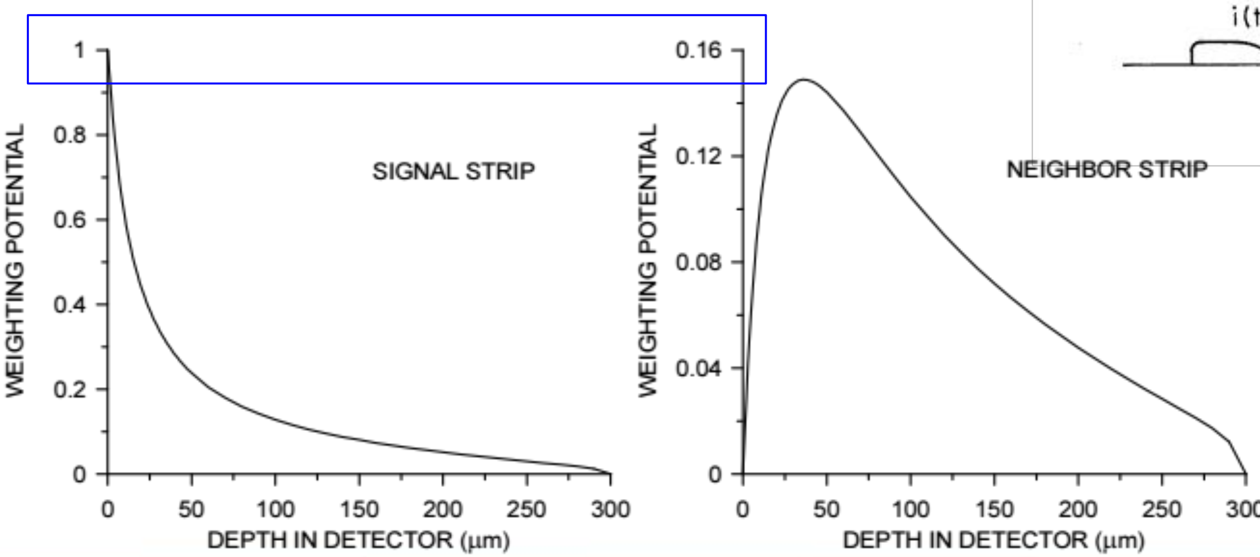
Pixelated TlBr - UM Orion Imaging Lab

Position-Sensitive Detectors - Lateral Position Resolution

- φ_w for one pixel extends laterally over neighbor as well
  - Resultant "transient" or "image" charge signals as basis for sub-pixel resolution

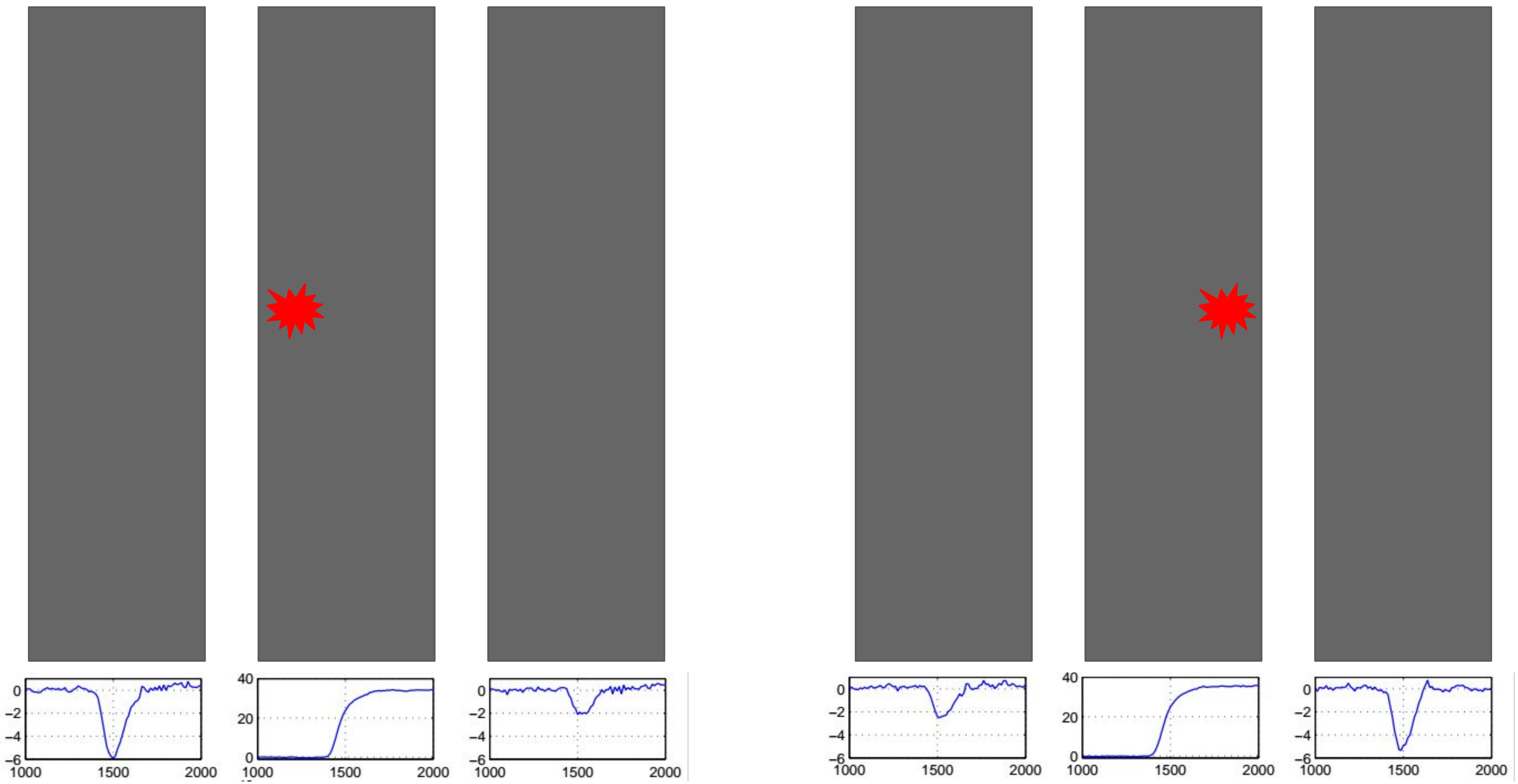


Radeka 1988 (RA 4)



Spieler 2.30

Sub-pixel Lateral Position Resolution



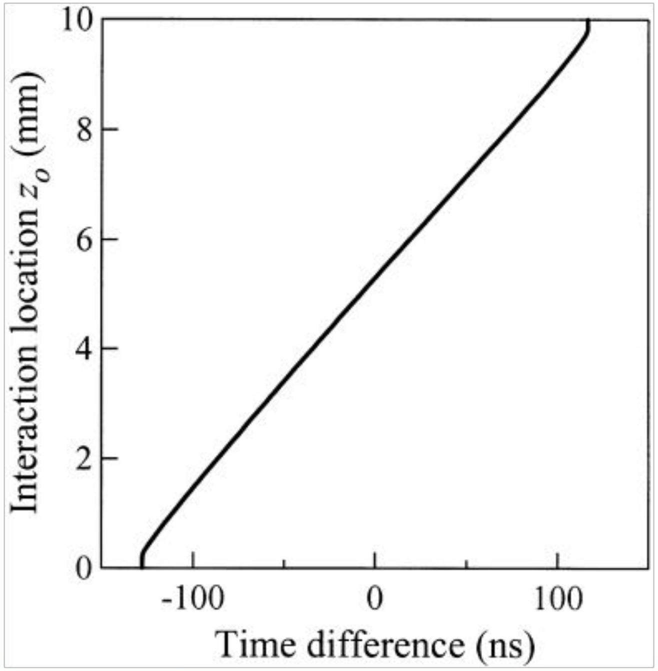
Signals From Thesis of RJ Cooper - c.f. smartPET

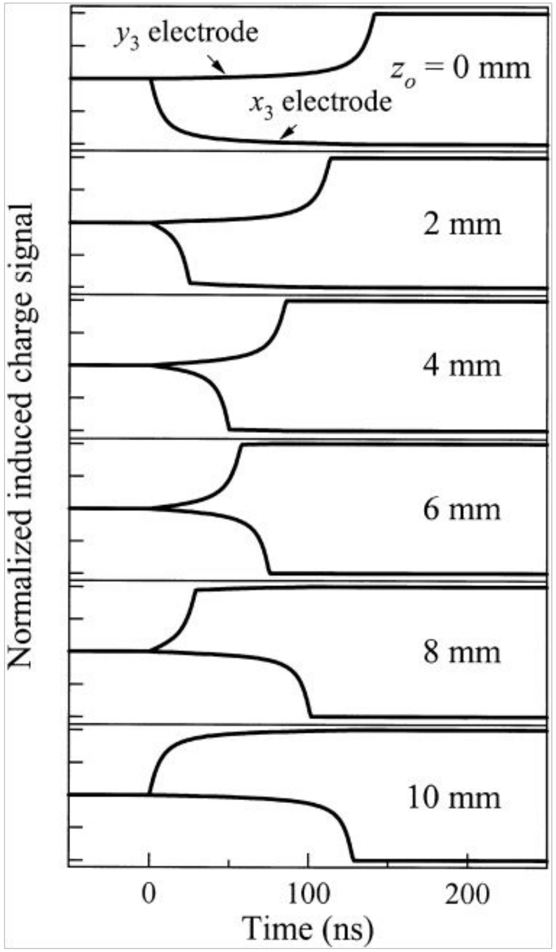
Position Determination - Depth

- Small pixel effect also provides means of recovering the depth of gamma-ray interactions
  - Not directly though - must be reconstructed by some means
  - Nature of depth sensitivity depends on carrier collection e.g.
    - DSSD - both carriers collected: Depth $\propto \Delta$(time-of-charge collection)
    - Single-polarity schema - Rely on amplitude or timing of cathode/anode signals

Depth of Interaction in DSSDs - ΔT_{50}

- DSSD detectors - both electrodes segmented = small pixel effect on each electrode
  - Maximum induced current (t_{50}) occurs very near the strip - treat the max current time as "arrival time" of charge cloud at strip

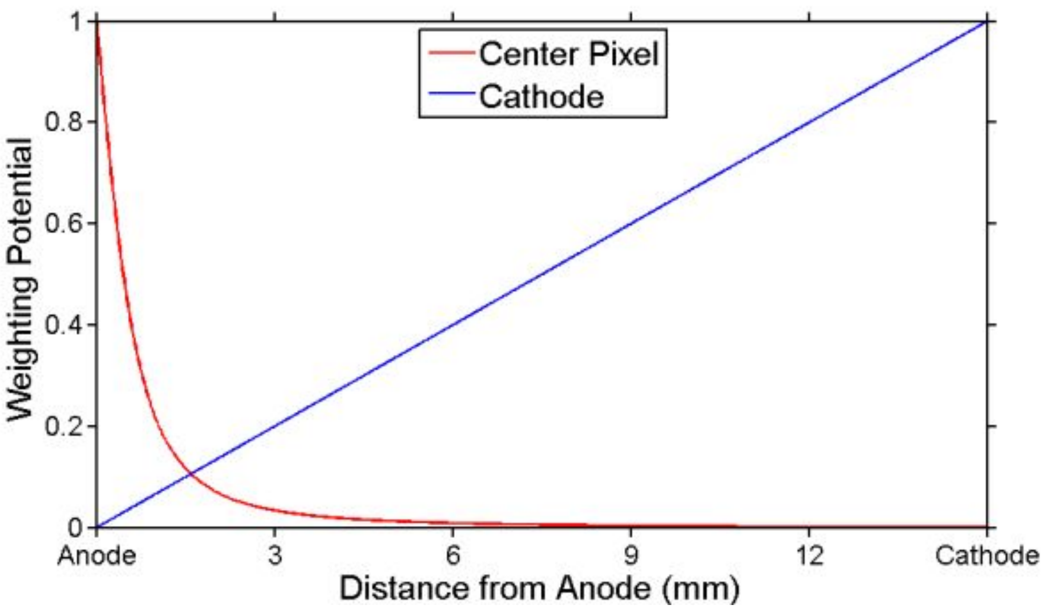


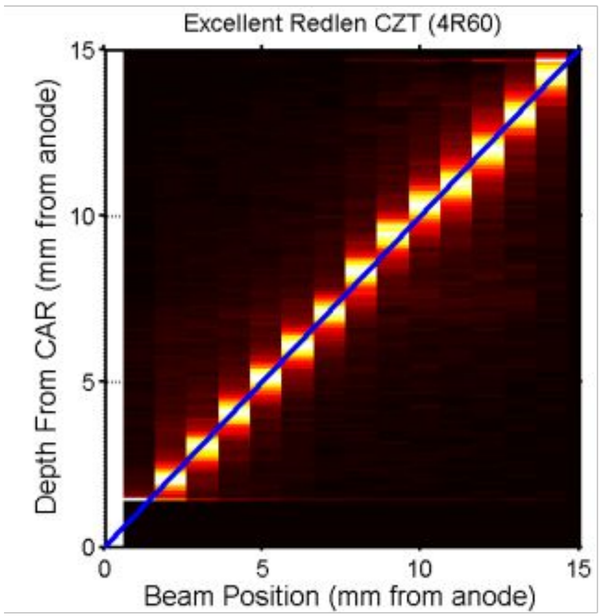


[Amman, Luke - Three-dimensional position sensing and field shaping in orthogonal-strip germanium gamma-ray detectors]

Depth of Interaction - Unipolar Sensing

- In instruments with poor μ_{hole} (e.g. CZT) can rely on relationship between cathode & anode signal
- For detectors with an unsegmented cathode (CPG, pix. anode)
  - Amplitude-based: Cathode/anode ratio
  - Electron drift time

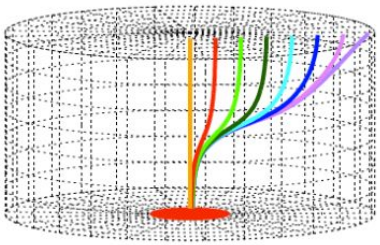


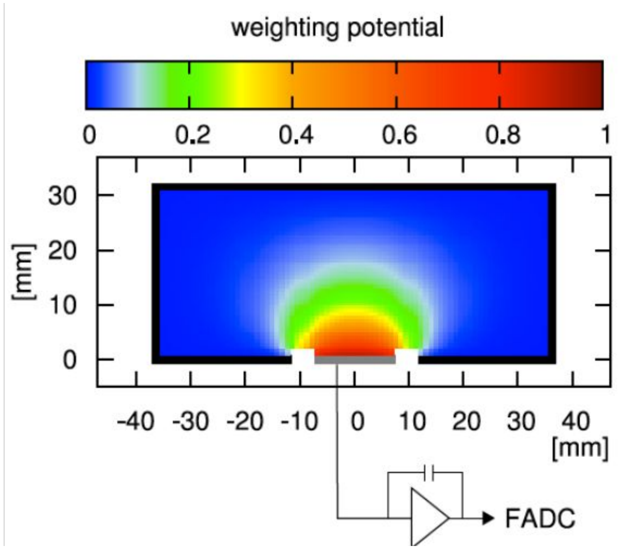


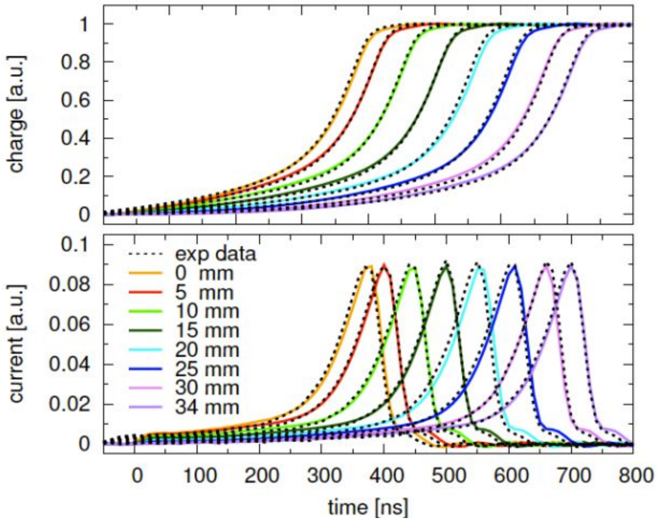
[W. Kaye - Energy and Position Reconstruction in Pixelated CdZnTe Detectors]

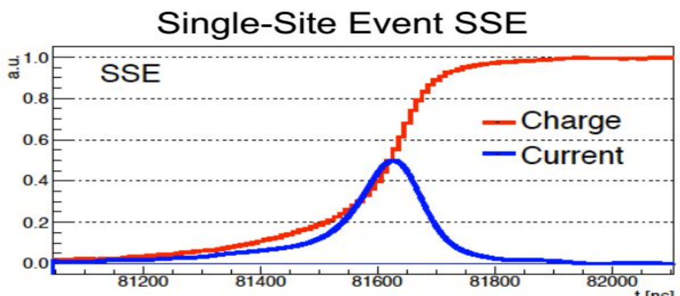
Pulse Shape Discrimination (in HPGe!)

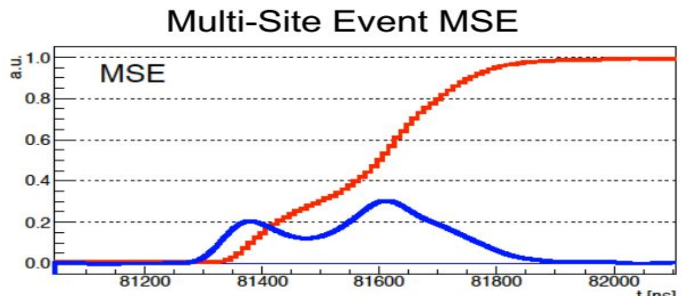
- Event selection based on pulse shape
  - E.g. Lab 2... or rare event searches (Majorana)
  - P-type Point-Contact (PPC) detector











Images from M. Agostini - Pulse Shape Discrimination for GERDA Phase I Data