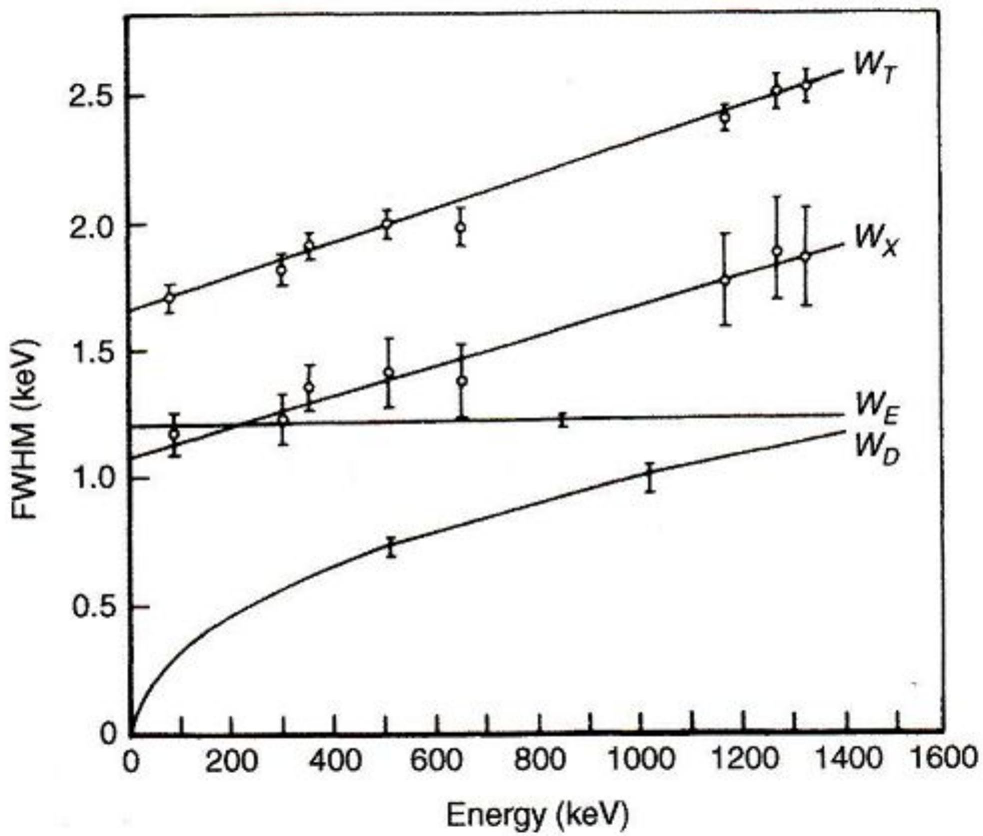
Attribution & Literature

The material for this lecture is derived from Spieler, particularly sections II, III, and IV. For the most complete treatment, consult Spieler's textbook Semiconductor Detector Systems

Energy Resolution

$$(\Delta E_{total})^2 = (\Delta E_{stat})^2 + (\Delta E_{electronics})^2 + (\Delta E_{charge\text{-}loss})^2$$

N.B. - More generally, shape of peak given by convolution of distributions describing each of these components!



Knoll 12.11

- Multiple elements contribute to width of peaks in energy spectrum
  - Statistics of carrier generation
  - Electronic Noise
  - Charge collection

Energy Resolution - Statistics of Carrier Generation

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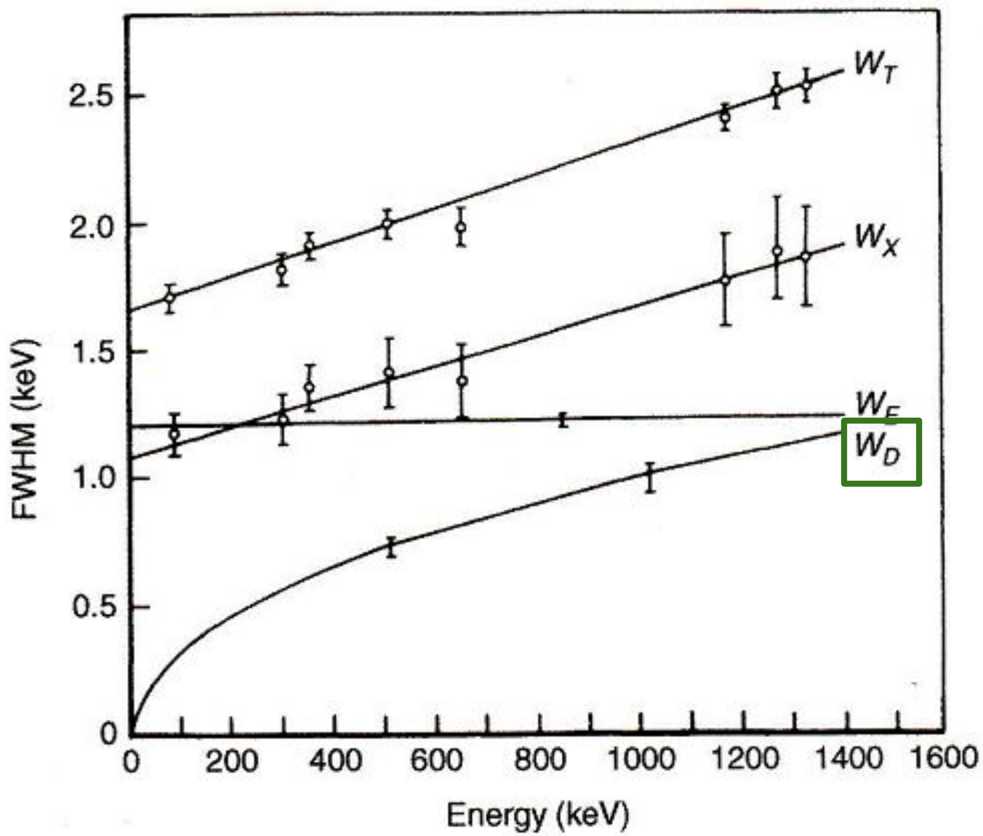

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Statistics of Carrier Generation

- Ionization spectrometers based on collection of *N* carriers generated by energy deposition *E*
- Carrier generation is a **stochastic process**
  - Let ε represent average energy required to generate an "information carrier", IC
    - **Semiconductors:** IC = e/hole pairs | ε = 3-5 eV
    - **Gas Ionization:** IC = e/ion pairs | ε ~ 30 eV
    - **Scintillation:** IC = optical photons | ε^* ~ 10 eV
      - When accounting for collection/photocathode efficiency, ε ~ 100 eV
- The generation of charge carriers is often modelled with the **Poisson Distribution**

Statistics of Carrier Generation

- **Poisson model** has nice property where *var(N)* = $\sigma_N^2 = N$
- However, model only applicable if carrier generation is **independent** of all other carrier generation events
  - Good model for scintillators: many competing decay modes
    - More detail in scintillator lectures
  - For gas ionization and semiconductors, ionization products are measured directly; limited number of mechanisms for energy absorption
    - Results in measured variance **less** than that predicted by Poisson model
- Deviation from Poisson-predicted variance quantified in **Fano Factor, F = *var(N) / N***
  - $F_{semiconductor} \sim 0.05 - 0.1$, $F_{gas\ (ionization)} \sim 0.2$
  - $F_{scintillator} \sim 1.0$

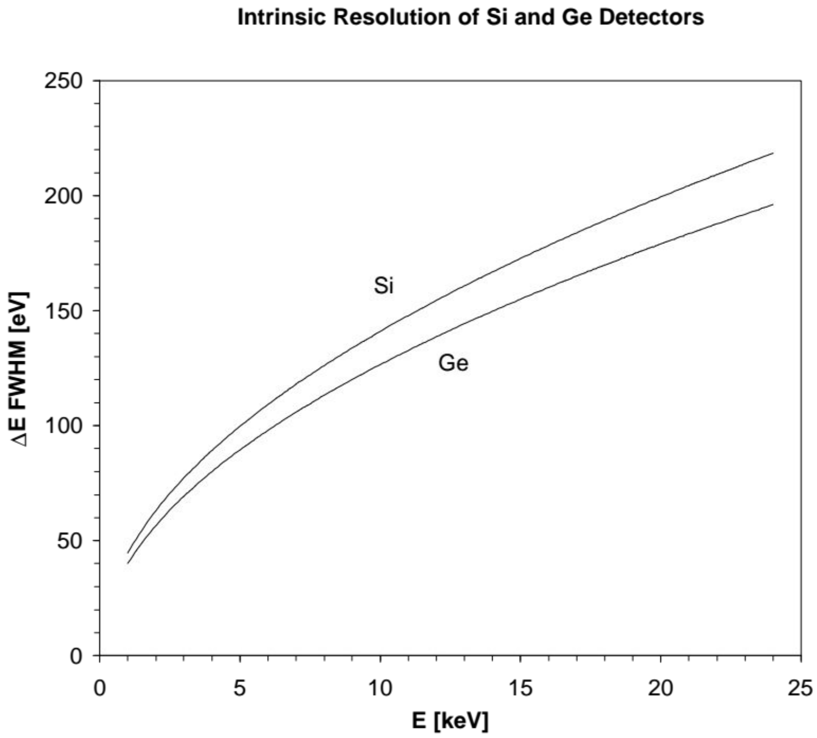
Fano Factor in Semiconductor Detectors

- Spieler provides concise, conceptual model for the origin of the Fano factor in semiconductor detectors
  - For more information, start with the original paper from Fano

$$\Delta E = 2.35 \cdot \varepsilon_i \sqrt{F N_Q} = 2.35 \cdot \varepsilon_i \sqrt{F \frac{E}{w}} = \boxed{2.35 \cdot \sqrt{F E \varepsilon_i}}$$

Si:	$\varepsilon_i = 3.6$ eV	$F = 0.1$
Ge:	$\varepsilon_i = 2.9$ eV	$F = 0.1$

- Spieler 2.2.3



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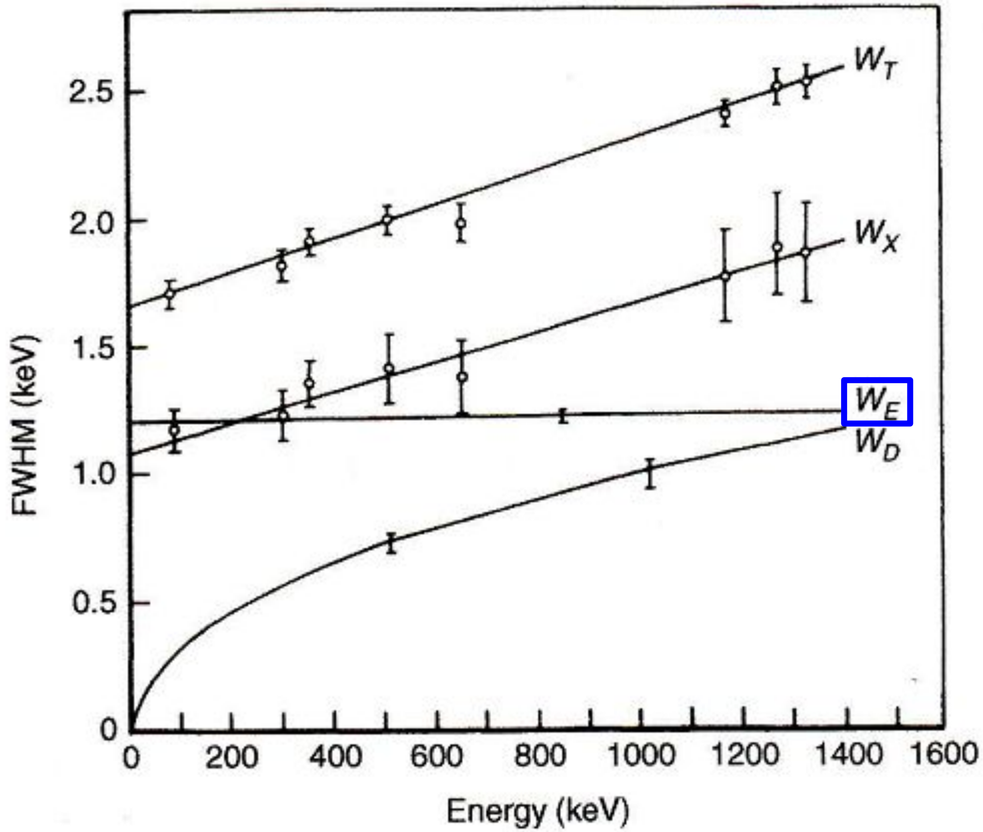

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Sources of Electronic Noise

- Basic mechanisms contributing to electronic noise (from Spieler)

Charges — Carrier velocity — Current (signal) — Electrode coupling (Shockley-Ramo)

$$i = \frac{n\, e\, v}{l}$$

$$\longrightarrow \quad \langle di \rangle^2 = \left(\frac{ne}{l}\langle dv \rangle\right)^2 + \left(\frac{ev}{l}\langle dn \rangle\right)^2$$

- Current fluctuations given by fluctuations in **number of charge carriers** and **charge carrier velocity**
  - **Velocity fluctuations** →thermal noise
  - **Number fluctuations**
    - Shot noise, e.g. current flow through barrier junction
    - Carrier trapping/detrapping →1/f noise

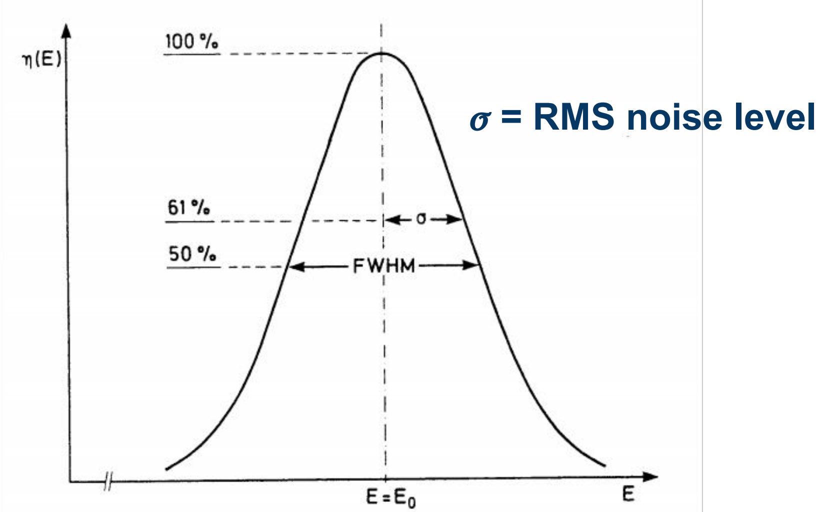
Quantifying Electronic Noise

- Describe noise in terms of spectral density, i.e. noise power per unit bandwidth
  - Spectral noise power density $\frac{dP_n}{df}$
  - Spectral noise voltage density $\frac{dv_n^2}{df} = \frac{dP_n}{df} R$
  - Spectral noise current density $\frac{di_n^2}{df} = \frac{dP_n}{df} \frac{1}{R}$
- Quantifying noise in terms of detector signal: **Equivalent Noise Charge**
  - Signal charge that yields SNR = 1
  - For ionization detector with average ionization energy, ε_i, noise level can be expressed in terms of energy by

$$E_{noise}(eV) = \varepsilon_i \cdot ENC$$

Characteristics of Electronic Noise

- Spectral distribution of both **thermal** and **shot** is constant
  - I.e. "white" noise: $\frac{dP_{noise}}{df} = const.$
- 1/f noise exhibits a frequency dependence: $\frac{dP_{noise}}{df} = \frac{1}{f^{\mathbf{a}}}$
  - ***a*** ~ 0.5 - 2 (discussed in a few slides)
- Thermal and shot noise are purely random and uncorrelated
- Amplitude distribution is Gaussian - deviations symmetric about DC baseline level



Spieler pg. 15

Thermal (Johnson) Noise

- Electron velocities given by thermal distribution
- Spectral density of noise power can be derived from long-wavelength approx. to blackbody spectrum (Spieler 3.4.1)

$$\frac{dP_n}{df} = 4kT$$

- In elements with finite resistance, gives rise to voltage/current fluctuations:
  - Spectral noise voltage density: $$\frac{dv_n^2}{df} \equiv e_n^2 = 4kTR$$
  - Spectral noise current density: $$\frac{di_n^2}{df} \equiv i_n^2 = \frac{4kT}{R}$$

Shot Noise

- Carrier injection over some potential barrier
  - Carrier injection across rectifying (PN junction), blocking contacts
  - Thermionic carrier generation (bulk leakage)
- Carrier injection an independent, stochastic process
  - Subject to statistical fluctuations
- Injection is random in time, contribution from each injection can be treated as delta pulse, yielding white (freq. independent) spectrum (see Spieler 3.4.2)

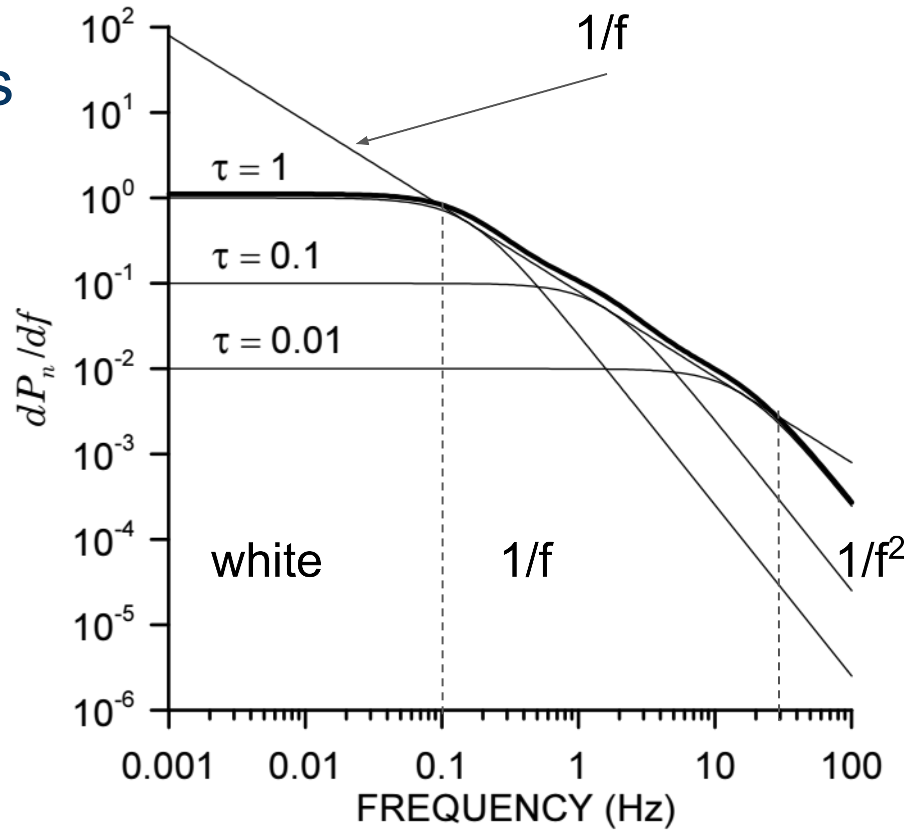
  - Spectral current noise density: $$i_n^2 \equiv \frac{di_n^2}{df} = 2eI$$

I = average current, *Ne*

1/f Noise

- Results from trapping/de-trapping of carriers
  - Trapping events are independent, random in time
- Characteristic times involved with traps of various depths
  - E.g. “shallow” trap→small τ, “deeper” trap →longer τ
- Multiple time constants give rise to 1/f behavior
  - Individual traps ~ 1/f^2

$$i_{nf}^2 = 4NI^2\left(\frac{\Delta G}{G}\right)^2 \frac{\tau}{1+(\omega\tau)^2}$$



$$\frac{dP_{noise}}{df} = \frac{1}{f^{\boldsymbol{a}}}$$

Where ***a*** ~ 0.5 - 2

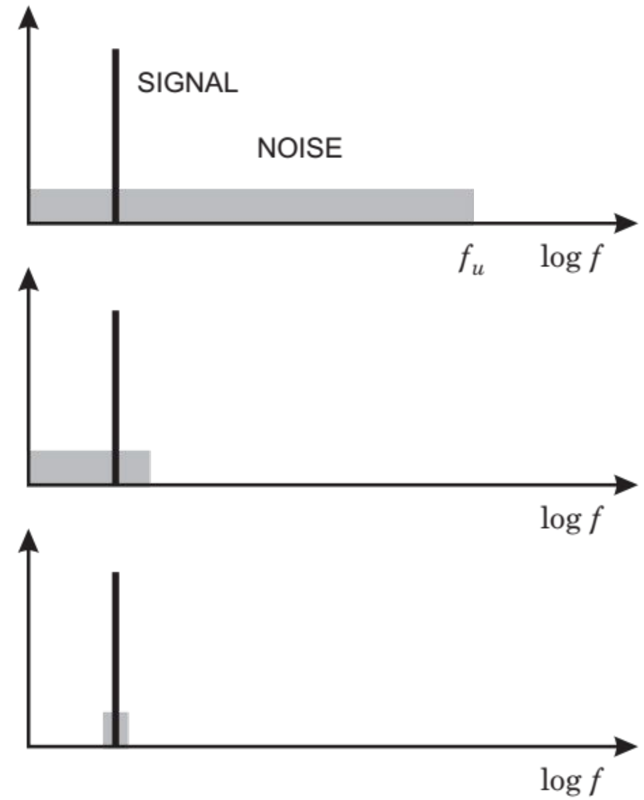
Signal Shaping

- Total noise of the system given by integrating spectral noise distribution over the bandwidth of the shaper

$$v_{no}^2 = \int_0^\infty e_n^2 A^2(f)df \quad \text{or} \quad i_{no}^2 = \int_0^\infty i_n^2 A^2(f)df$$

  - v_{no}, i_{no} = noise at output of shaper
  - *A(f)* = gain of shaper

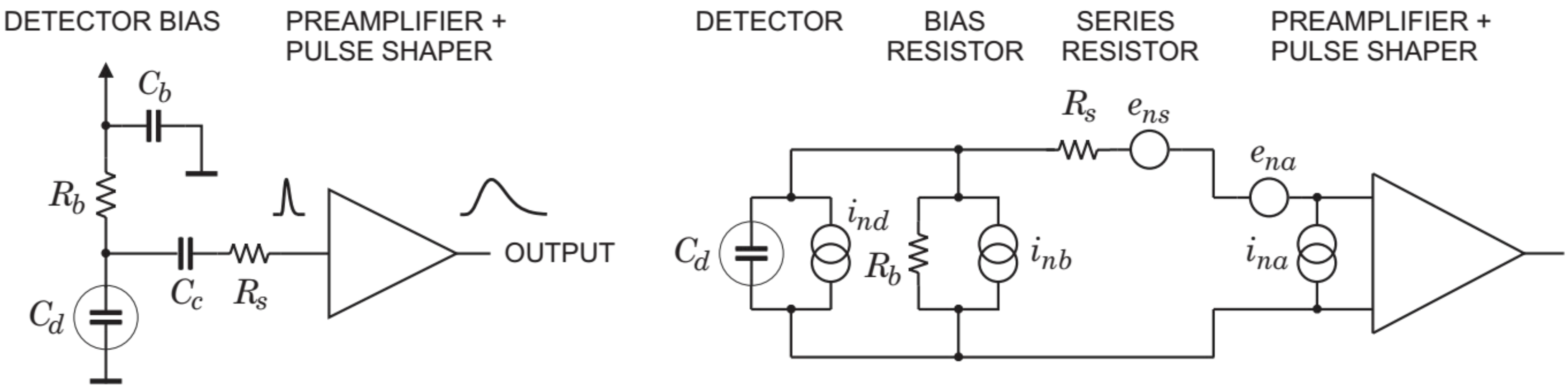
- Total noise increases with $(BW)^{1/2}$
- **N.B.** Decreasing BW →longer rise times



Spieler fig 3.3

ENC of Charge-Sensitive Front End

Model Ckt of signal chain | Equiv. Ckt for analytic noise analysis



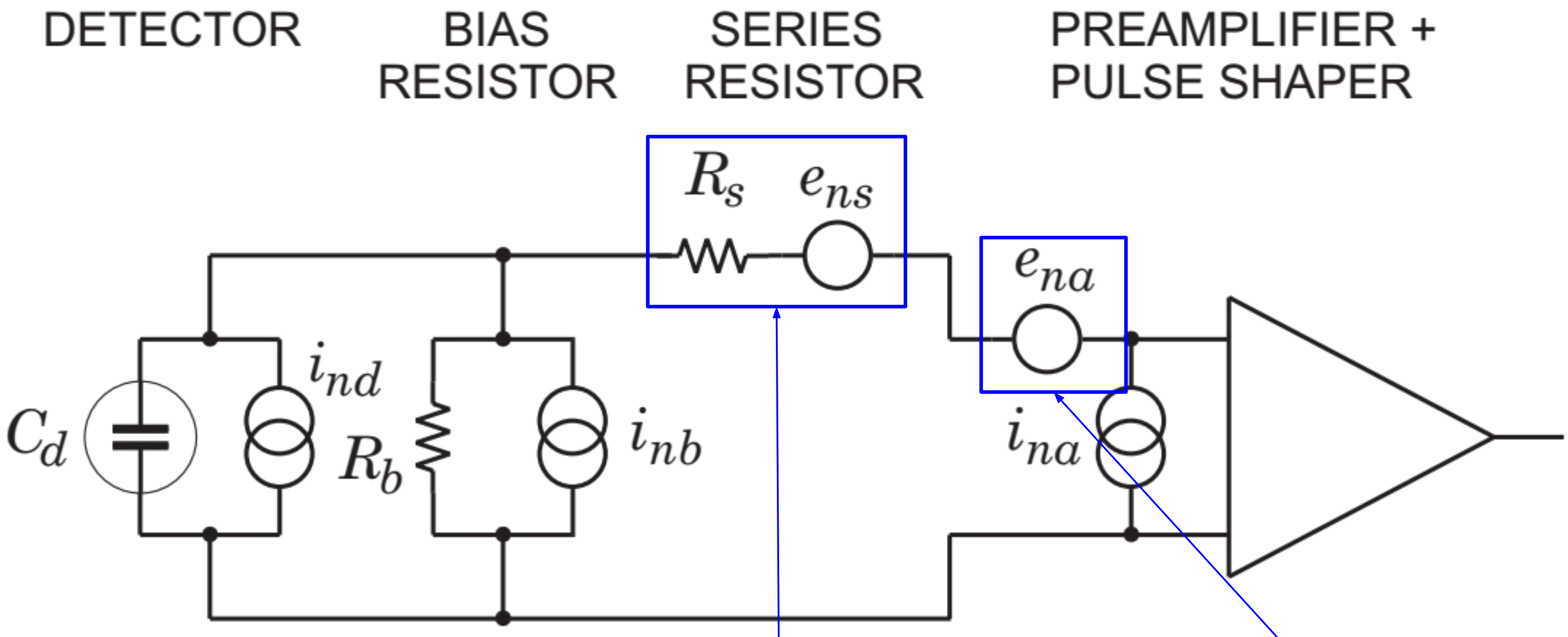
Spieler fig 4.7

N.B. - Resistors can be modelled as voltage or current sources

- Resistors in **parallel** with input: noise current sources
- Resistors in **series** with input: noise voltage sources

This is where the "series" / "parallel" noise monikers come from

Voltage (Series) Noise Sources

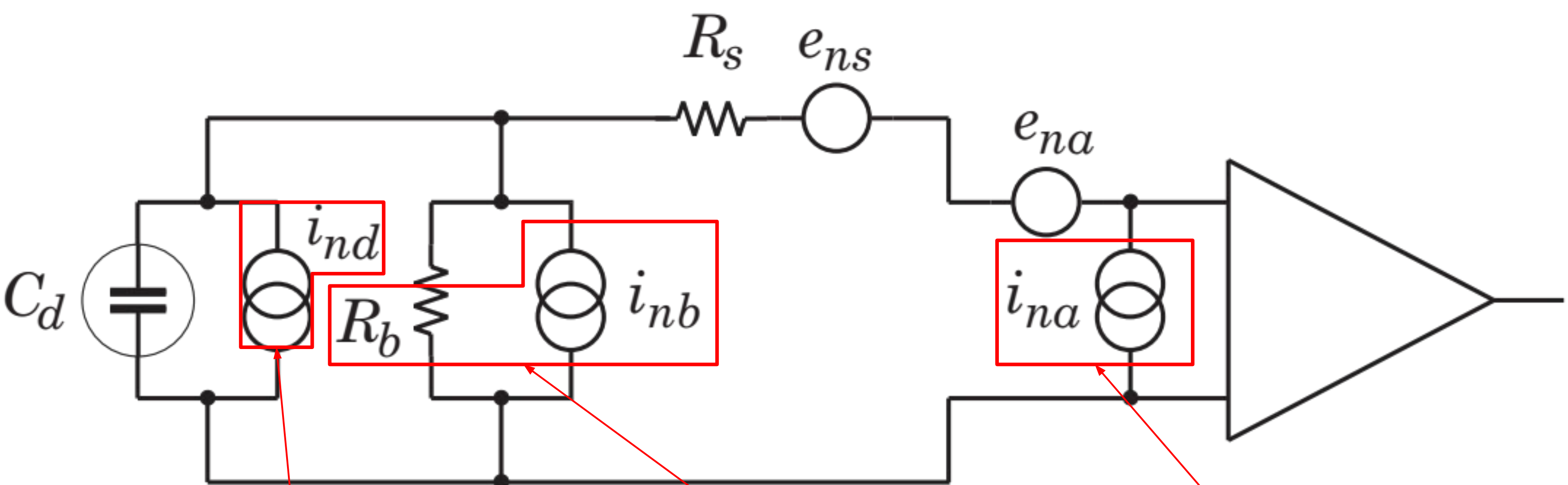


$R_{electrodes}$, R_{wires} → thermal noise
$e_n = 4kTR_s$

V noise, amp. Input (white & 1/f)

Current (Parallel) Noise Sources

DETECTOR | BIAS RESISTOR | SERIES RESISTOR | PREAMPLIFIER + PULSE SHAPER



Shot noise (e.g. leakage current)

$$e_{nd}^2 = i_{nd}^2 \frac{1}{(\omega C_d)^2} = 2eI_d \frac{1}{(\omega C_d)^2}$$

Thermal noise in R_b

$$e_{np}^2 = \frac{4kT}{R_p}\left(\frac{R_p \cdot (-\mathbf{i}/\omega C_d)}{R_p + (-\mathbf{i}/\omega C_d)}\right)^2$$

Shot noise in preamp input FET

$$\frac{i_{na}^2}{(\omega C_d)^2}$$

Current (Parallel) Noise Sources

- Noise from feedback resistor
  - Thermal (Johnson) noise in feedback resistor parallel to amplifier input
- Very low-noise front-ends try to reduce noise by moving away from resistive feedback to eliminate this noise source
  - Leakage current through diodes in TRPs are a noise source too!
    - Optical-reset CSA's

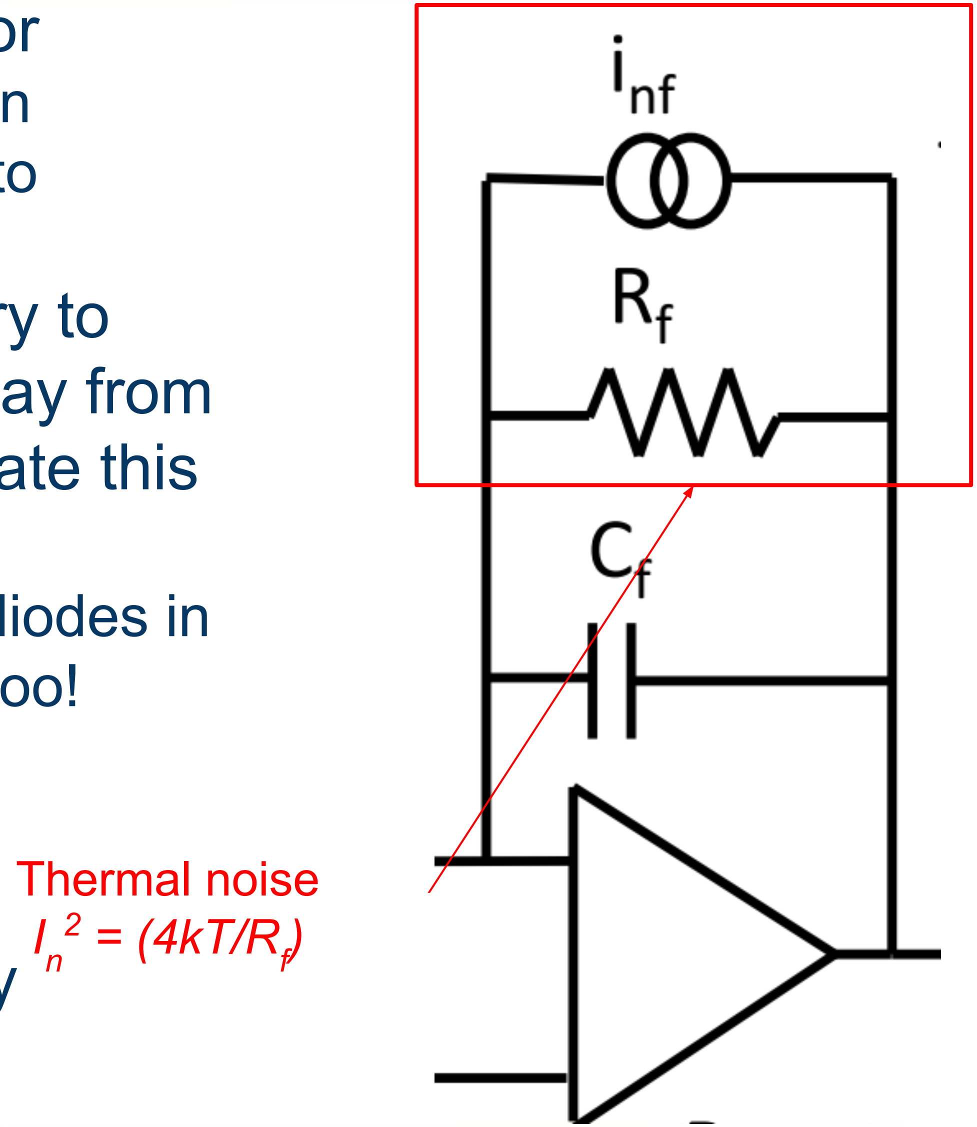


Figure created by Brian Plimley

ENC Analysis

Cumulative input noise voltage

$$e_{ni}^2(f) = e_{nd}^2 + e_{np}^2 + e_{nr}^2 + e_{na}^2 =$$
$$= \frac{2eI_d}{(\omega C_d)^2} + \frac{4kTR_p}{1+(\omega R_p C_d)^2} + 4kTR_s + e_{na} + \frac{i_{na}^2}{(\omega C_d)^2}$$

Noise voltage at output depends on freq. resp. of amplifier

$$V_{no}^2 = \int_0^\infty e_{no}^2(f)df = \int_0^\infty e_{ni}^2(f)\left|A_v\right|^2 df$$

Specific example of **ENC for CR-RC** shaper with $\tau_{int} = \tau_{diff}$

$$Q_n^2 = \left(\frac{\epsilon^2}{8}\right)\left[\left(2eI_d + \frac{4kT}{R_P} + i_{na}^2\right)\cdot\tau + \left(4kTR_S + e_{na}^2\right)\cdot\frac{C^2}{\tau} + 4A_f C^2\right]$$

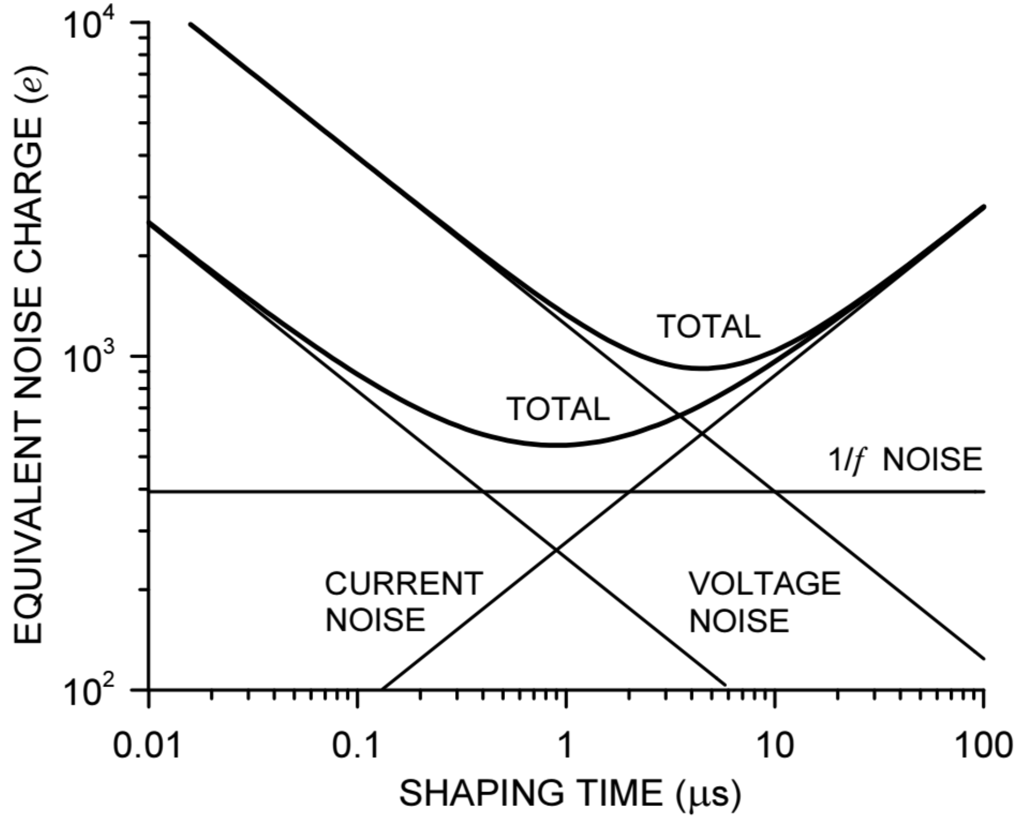
See Spieler 4.3.5 for full treatment

ENC Analysis

A more general result for ENC was derived in RA 4 (Radeka):

$$Q_n^2 = i_n^2 F_i T_S + e_n^2 F_v \frac{C^2}{T_S} + F_{vf} A_f C^2$$

- *T_s = shaping time*
- *C = total input capacitance*
- *F_i, F_v, F_{vf} = "Shape factors"* - can be computed from IR of shaper (see Spieler 4.5.2)



Spieler fig 4.29

ENC Analysis

A more general result for ENC analysis was derived in reading assignment 4:

$$Q_n^2 = \boxed{i_n^2 F_i T_S} + \boxed{e_n^2 F_v \frac{C^2}{T_S}} + \boxed{F_{vf} A_f C^2}$$

Current noise sources

- Independent of input capacitance
- Contribution increases with increasing T_s

Voltage noise sources

- Increases rapidly with input capacitance
- Contribution decreases with increasing T_s

1/f noise sources

- Increases rapidly with input capacitance
- Independent of T_s, but depends on BW

$$v_n^2 = \int_{f_l}^{f_u} \frac{A_f}{f} df = A_f \log \frac{f_u}{f_l}$$

Minimum noise:

$$Q_n^2 = 2 e_n i_n C \sqrt{F_i F_v} + F_{vf} A_f C^2$$

Note dependence on shape factors!

Noise Curves from Real Systems

- Some examples in Spieler 4.4
- Noise curve from one of our HPGe's - can you diagnose the problem?

Shaping TIme	FWHM (keV) @ 2402.57 keV, HV = +2400	FWHM @ 2402 HV = +2500
12	0.75	4.6
6	0.69	2.6
4	0.74	2.05
2	0.85	1.33
1	1.01	1.11
0.5	1.27	1.27